DOCUMENT RESUME

ED 283 672 SE 048 207

AUTHOR	Vonder Embse, Charles Bernard, Jr.
TITLE	An Eye Fixation Study of Time Factors Comparing Experts and Novices When Reading and Interpreting Mathematical Graphs.
PUB DATE	87
NOTE	154p.; Ph.D. Dissertation, The Ohio State University.
PUB TYPE	Dissertations/Theses - Doctoral Dissertations (041)
EDRS PRICE	MF01/PC07 Plus Postage.
DESCRIPTORS	*College Mathematics; *Data Interpretation; *Eye Fixations; Eye Movements; *Graphs; Higher Education; Mathematics Curriculum; Mathematics Education; *Mathematics Instruction; *Mathematics Skills; Visual Aids; Visual Perception
IDENTIFIERS	*Graphing (Mathematics)

ABSTRACT

A graph is an example of a symbol system which encodes information through the juxtaposition of marks in a two dimensional plane; decoding the meaning of a graph uses specific attributes of the human visual system. This study was undertaken to identify some variables critical to graph interpretation as a first step in designing better curriculum materials and methods of instruction. The study was developed to define and investigate the differences between novice and expert graph readers on the dimensions of average fixation duration, percent of total viewing time, and performance on a selection and memory task. Results indicated that the experience of the graph reader played a major role in the interpretation process by signaling the need for the cognitive decision to increase fixation duration in important blocks of the graph. The mathematical graphs used in the study tended to draw experts' as well as novices' eyes to the important information. (TW)

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ED283672

AN EYE FIXATION STUDY OF TIME FACTORS COMPARING EXPERTS AND NOVICES WHEN READING AND INTERPRETING MATHEMATICAL GRAPHS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

Charles Bernard Vonder Embse, Jr., B.A., M.A.

* * * * *

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To My Wife, Shirley

ACKNOWLEDGEMENTS

I extend my heartfelt thanks to Dr. Alan Osborne for his help and guidance in this project and throughout my doctoral program. My thanks go to Dr. Elizabeth Bernhardt whose initiative made the eye movement laboratory a reality. Thanks go to Dr. Suzanne Damarin whose guidance and council has assisted me throughout my doctoral program. Very special thanks go to Dr. Bert Waits for his belief in me and his encouragement and guidance throughout my master's and doctoral program.

Words seem totally inadequate to express my deepest thanks to my wife, Shirley. Without her help and encouragement, my doctoral program would not have been possible. And to my children, Beth and Susie, thank you for being so understanding when I could not give you the time you deserved.

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CHAPTER I

INTRODUCTION

A graph is an example of a symbol system which encodes information through the juxtaposition of marks in a two dimensional plane. Unlike text which uses a fixed set of symbols, graphs employ various geometric shapes to encode meaning. Interpretation of a graph is a visual task. Decoding the meaning of a graph takes advantage of the human visual system and its ability to interpret shape, size, texture, color, density, and spatial relationships. Working in harmony with the cognitive structure of the graph reader, the visual system acquires and transmits information to the brain and receives feedback from the brain to regulate the viewing process.

The study of eye movements in interpreting graphs is a new area of research. Eye movement data provides direct empirical evidence about the cognitive process of decoding a graph. This research study used eye tracking equipment to gather data on the position and duration of eye fixations when viewing mathematical graphs. Findings of this study will help define important variables and build an understanding of the cognitive process of decoding a graph. This is the first step to a more complete understanding of the learning and teaching process involving graphing skills; these areas have proven to be deficient in the past, but promise to be of ever increasing importance in the future.

1.1 The Importance of Graphs and Graphing Skills

Graphs and graphing skills are an important part of the mathematics curriculum. The National Council of Supervisors of Mathematics identified graphing skills as one of ten basic mathematical skills necessary for all students (NCSM, 1977). School curriculums at all levels use graphs as a vehicle for display and organization of data, in concept development, and in problem solving. Computer generated graphs in the classroom are becoming more common as hardware and software innovations reach the mass market. Graphs of various types are used in most fields of study to help organize data and information, and to present it more clearly. Graphs can be a compact, understandable, elegant tool for conveying the complex relationships between variables.

1.2 The Problem

Research has indicated that students' graphing skills are not developed beyond the basics of point plotting. The First, Second, and Third National Assessments of Educational Progress (Carpenter, 1975, 1980, 1983, NAEP, 1979) identified students' graphing skills as "superficial." Students were able to read simple graphs, but could not perform related skills such as interpreting, generalizing, integrating, or extending the information in the graph. The Second International Mathematics Study (SIMS) contained 23 items concerned with graphing and analytic geometry. Pretest and posttest scores for twelfth grade students enrolled in precalculus courses were 34% and 43% respectively (Demana & Waits, 1987, and Travers, 1985). Kerslake (1977) found that most 13 - 15 year olds could read graphs and plot points on a grid system, but were not as successful when it came to interpolation using decimals, dealing with questions of slope and rate of change, or understanding the relationship between a graph and its equation (Hart, 1980). Bell and Janvier (1981) found that most instruction in graphing deals with point reading and some comparison of graphs, but does not treat the global features of graphs such as general shape, intervals of rise and fall, and maximum increase and decrease. The National Council of Teachers of Mathematics, in An Agenda for Action: Recommendations for School Mathematics of the 1980's, recommended "increased emphasis" on higher level graphing skills such as organizing and presenting data and graphical models in problem solving as an important goal for this decade (NCTM, 1980).

The present school curriculum is not preparing students with the graphing skills that they will need in the future. This fact holds true for both those students who go on to higher education as well as those who go directly into the work force. Evidence from the three NAEP studies and Second International Mathematics Study shows that schools need to do a better job of teaching graphing skills. The need for training in these skills for both college and non-college bound students is clear. We are in the midst of an information explosion. The computer age has brought increased demands on all persons to absorb and understand vast quantities of information. Graphs and graphical displays are the most efficient method of encoding large quantities of information in easily understood and compact ways.

The question of "Why aren't the schools doing a better job of teaching graphing skills?" is legitimate. The answer is complex. It involves the content of the curriculum, placement of topics in the curriculum, methods of instruction, and teacher training. But, all of these concerns hinge on a more basic understanding of what it means to interpret a graph and understand the information encoded in the graph. More specifically, what are some of the critical variables which define the process of graphical interpretation? Once these critical variables are defined and understood, better answers to the previous curriculum and teaching questions can be formulated. Only then can effective changes be implemented in the curriculum.

1.3 Definitions

For this discussion, graphs will be partitioned into two broad categories: mathematical graphs and data graphs. Mathematical graphs are those graphs which represent a functional or mathematical relationship between two or more variables. For example, the graph of the functional relationship "$f(x) = 2X + 3$"

is a mathematical graph. Data graphs are graphs which display an empirical data set describing one or more variables. An example of a data graph would be a bar graph showing the number of students enrolled in various classes at a school. Both types of graphs require that the user have a set of basic skills for the interpretation and decoding of the information embedded in the graph. These basic skills are generally the same for both types of graphs. They involve the ability to read, interpret, integrate, and extend the information in the graphs.

Graphs display physical features and relationships. The physical features include the type of graph, scales, amount and placement of information, slope, general shape, rate of change, maximum/minimum values, color, density, background, dimensionality, continuity, density, and other attributes. Simcox (1981) refers to these attributes as component properties of the display. There are also holistic properties of the display which arise from the interaction and interrelationships between the component properties. Together the component and holistic properties make up the encoding features which graph readers use to represent the display mentally.

Interpreting a graph means the cognitive decoding of the visual information transmitted to the brain. This information comes from the physical details and interrelationships of the graph through interpreting, integrating, generalizing, and extending the information and relationships of the graph. The cognitive decoding process used by an individual to process the incoming information can only be inferred from the external evidence. We cannot peer into a subject's brain to examine the actual cognitive process employed. Several competing theories of cognitive processing have been proposed. These theories have been used to explain aspects of the decoding process.

1.4 Graphical Interpretation Theories

The way in which data is encoded into the graphical display itself is hypothesized to be closely related to the interpretation of the graph. Tufte (1983) formulated a theory of data graphics that attempts to organize the process of encoding data and show better ways to convey meaning through graphs. His theory addressed both the practical factors like data-ink ratios, chart junk clutter, and lie-factor ratios as well as the aesthetics and elegance of data presentation. Cleveland and McGill (1985) developed a theory of graphical perception which also dealt with the encoding of data in graphical displays. They have proposed a rank ordering of elementary graphical perception tasks which predict the amount of error in the perception of graphical details. Pinker (1981) proposed a theory of graphical perception related to more global interpretation of graphs. His hypotheses stated that graphs can convey information effectively because "...they can display global trends as geometric patterns that our visual systems encode easily" (Pinker, 1983). He concluded that the graphical formats used and the kinds of information conveyed by graphs are not equally difficult. Certain types of information are conveyed more efficiently in certain graphical formats according to naturally perceivable visual patterns within the graph itself. Pinker's model dealt with more general graph comprehension than the elementary perceptual tasks model of Cleveland and McGill.

Two foci for the interpretation of graphs have now been identified: the characteristics of the viewer and the display itself. When interpreting a graph, these interact in many ways. Individual differences in background, experience, and previous knowledge affect the interpretation of a graph. Graphs representing the same data, but in slightly different versions, can be interpreted in different ways by the same viewer. This multidimensional interaction of factors can cause problems when attempting to investigate causation between the physical features of the graph and the interpretation of a graph. Theories about how physical details make a graph understandable help show that the physical features of the graph can be controlled. The variable factors are the visual processing and interpretation by a human subject.

1.5 Appropriateness of Studying Graphs

While many researchers have investigated eye movement patterns in the context of reading text and viewing pictures and natural scenes, there is no research which applies the methodology of eye movement research to reading mathematical graphs or data graphs. Graphs have many of the same characteristics as pictures in that they convey meaning through the presence and juxtaposition of geometric elements. Graphs may be more appropriate than pictures for investigating eye movement patterns since large amounts of relational information can be encoded with a minimum of actual markings. Norton and Stark's (1971) original work on the theory of "scanpaths" has been criticized on the grounds that the pictures they used were too simple to provide an information-rich viewing scene. Graphs can be very information rich and yet involve very few symbolic elements. Tufte (1983, p. 132) gives an example of a graph which maximized the data-ink ratio (see Figure 1). This graph represents a scatter plot of data. The limits of the axes show the maximum and minimum values obtained by each variable. The offset portions of the axes show the quartile ranges and the blank spot on each axis represents the median score for that variable. Ten extra pieces of information about the two variables in this example have been encoded by the elimination of ink from the graph. This is just one example of the parsimonious use of markings conveying a multitude of meaning.

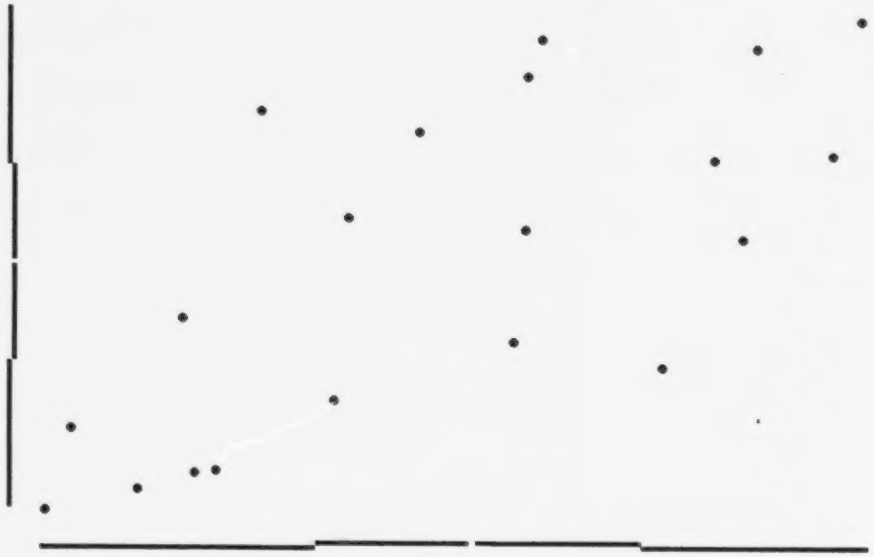


Figure 1: Scatter plot of data showing a high data-ink ratio (Tufte, 1983).

Eye movement research is an attempt to explain how meaning is processed cognitively. If the scene which is viewed is so simple that it is devoid of most meaning, then there is little processing occurring as that picture is viewed. If the picture viewed is so information rich that it is complicated, then discovering viewing patterns may be masked by other issues such as the subject's existing reference frames. If a picture contains many salient features, then the competition for attention between these features may also mask any viewing strategy used by the subjects. Viewing and interpreting mathematical graphs solves many of these problems. Graphs can be simple and information rich.

1.6 Research Technique

Information about the specific details attended to when interpreting a graph is usually obtained secondhand. The data is gathered from an interview during or after the actual act of viewing. Use of self-report interview data provides little or no information about the exact location and duration of the viewing scan. Direct eye movement data on the location and duration of the visual fixations can help researchers understand which features, relationships, and viewing patterns are important for correct interpretation of graphs. This study was performed using eye tracking equipment to define precise eye position, fixation duration, and total viewing time for the viewed graphs.

Eye movements are classified into two types, fixations and jumps. A fixation occurs when the eye comes to rest and focuses on a specific location in the field of view. A jump is the movement of the eye between fixations. Eye jumps are also called saccades (from the French meaning "to jerk on the reins of a horse"). Processing of new information occurs during fixations, but not during jumps (Wolverton and Zola, 1983). Eye position is the location of the center of a fixation. The surface area of the fixation is approximately a circular region with a diameter corresponding to 2 degrees of angular rotation of the eye (referred to as the foveal area) (Loftus, 1983). The surface area varies according to the distance from the eye to the object being viewed.

Fixation duration is the length of time between eye jumps (usually in milliseconds). Another way to define fixation duration is the length of time the eye remains focused on the same location before moving to another location. Fixation durations range from a minimum or 50 milliseconds to over 2000 milliseconds. The average fixation duration when reading text is between 125 and 250 milliseconds. A minimum of 100 to 125 milliseconds of fixation duration is needed to view an item and transmit that information to the brain (McConkie, 1983). Fixation durations in the range of 400 to 1000 milliseconds are considered long.

Total viewing time can be defined in two ways. First, it can be the simple total time spent viewing some passage or graph. Second, total viewing time can be broken down to represent the total viewing time spent in specific areas of the viewing field. For example, if the viewing field were divided into 100 equal square areas, then the total viewing time for each of the 100 areas could be calculated and studied. It is in this second sense that total viewing time was used in this research study.

In order to identify and investigate important variables, an expert versus novice study was done. Expert graph readers can and do use graphs effectively. They have experience using graphs to organize data, develop concepts, and solve problems. They have successfully acquired the needed graphing skill. Comparing experts to novices provides more specific information about what novices need to learn to become expert graph readers. This type of information is important for the educational goal of this type of research.

Eye tracking data are considered reliable indicators of cognitive processing and are relatively free from extraneous influences commonly related to experimental conditions. Graesser and Clark (1985) refer to the reliability of eye movement data:

> Some psychologists have argued that complex verbal protocols are not valid indices of comprehension. Many of these arguments are based on a "gut skepticism" rather than on data. One frequent complaint is that the act of articulating knowledge during comprehension interrupts or changes the normal course of comprehension. However, the same complaint could be raised about all of the simple response measures that researchers collect (except eye movement data). (p.10)

By controlling the graphs which are presented and the information given before viewing a graph, research on graphical interpretation using eye movement data can reveal relationships between the physical features viewed, the amount of time spent visually decoding the information in those features, and the patterns used in scanning the scene. This type of research was used to build the understanding of the basic graphical interpretation process needed for answering the curriculum and teaching questions.

1.7 Research Questions

The two specific dependent variables measured in this study were subjects' average fixation duration and total viewing time in certain areas of mathematical graphs. In light of the previous research and the documented deficiencies in student performance, this research study addressed these questions concerning the process of graphical interpretation:

1. Is there a difference between experts and novices in the total amount of time spent attending to different specific areas of mathematical graphs? Does the time spent in a distinct area of a graph represent a sustained amount of viewing time, or is it the collection of many short fixations in that area?
2. Is there a difference between experts and novices in their average fixation duration in different areas of a graph? Does one group of subjects have longer average fixation durations when viewing certain areas of a graph?
3. How does the total amount of time and the average fixation duration in distinct areas of the graph correlate between and within groups of subjects?
4. Does the symbol system (ie. the graph itself) draw the viewer to the important areas of the graph?

1.8 Summary

Eye tracking research on graphical interpretation is a new area of research. There were a large number of suggested questions about the relationship between graphical interpretation and several dependent variables. A pilot study was conducted to help establish which questions and variables would be studied first. The conclusion was that a study of the dichotomy between experts and novices would yield important information about the graphing skills needed by students. The nature of the eye tracking equipment allows questions about fixation time and eye position to be answered exactly. Evidence from the pilot study indicated that there were some basic differences between novices and experts in the total viewing time spent, and the average fixation duration in certain areas of mathematical graphs. A careful research study was then designed to ascertain if the observations made in the pilot study would maintain under tighter control and statistical analysis.

CHAPTER II

REVIEW OF THE LITERATURE

2.1 Introduction

Literature germane to this study of eye movements in graph interpretation can be separated into three areas:

1. Reading research using eye movement data
2. Research on viewing pictures and natural scenes using eye movement data
3. Graphical interpretation theories.

To this date, there is no research on graph interpretation using eye movement equipment. Reading research using eye movement equipment provides a basis for comparison of the findings of this study on reading graphs. The research on viewing pictures and natural scenes provides a counterpoint to the reading research relevant to this research because of the nature of graphs. Graphs share the spatial nature of pictures. Graphs are an example of a symbol system using the spatial relationships of geometric shapes to encode meaning. But graphs also share the symbolic nature of a text in that meaning is encoded using symbolic features rather than representations of natural features. Both areas of research literature help explain the process of reading graphs.

Several graphical interpretation theories have been proposed. These theories speculate on the cognitive processes involved in decoding a graph. Evidence to support these theories is not from eye movement data, but uses paper and pencil performance tasks. These theories provide some insights into the graph interpretation process.

2.2 Reading Research

Reading, in this discussion, is defined to be reading of text passages for the purpose of understanding and remembering the meaning encoded in the words. Research using methods of artificial constraints on the eye movements such as tachistoscopic presentations do not provide information on selective reading patterns in a "real world" context. This research will not be considered. If reading patterns are some type of reflection of mental processing, then realistic presentation of the stimulus is essential. Generalizability of research results is enhanced when the presentation of stimuli closely resembles the normal reading process being studied. This is true for reading graphs as well as text.

Reading research concerning eye fixations is organized around four factors:

1. fixation duration
2. fixation frequency
3. fixation location
4. fixation sequence

Fixation duration and fixation frequency are termed temporal factors since they involve elapsed time factors. Fixation location and fixation sequence are called spatial factors since they refer to the position of fixations, the distance between each fixation, and the sequential pattern of the fixations in the visual field. Fixation duration and frequency are the temporal factors which were the subject of this research study.

Carpenter and Just (1978) defined a relationship between these two time factors. Gaze duration is the total time spent looking at a word in a text without regard to the number of individual fixations. The length of a gaze duration is found by summing the durations of all the fixations occurring on a given word. For example, if the word "horse" in a text passage was processed with three fixations of 175, 220, and 190 milliseconds, the gaze duration would be 585 milliseconds.

The two dependent variables defined in this study were average fixation duration and percent of total time spent in a given area of a graph. Average fixation duration was the arithmetic mean of the duration of all the fixations occurring within a specific area. Percent of total time for a specific area of a graph was similar to gaze duration for a single word in a text passage. Percent of total time, like gaze duration, showed the sum of the durations of all the fixations in a specific area. But, the percent of total time figure also gave information about how the fixation frequency in one area compared to all the other areas of the graph by representing a percent of the grand total of viewing time for the entire graph.

2.3 Eye Movement Control in Reading

Two differing theories concerning the control of temporal and spatial eye movements patterns were suggested. The "global" theory proposed that eye movement patterns were controlled by a global strategy established before reading began. Bouma and deVoogd (1974) proposed that eye fixation patterns were independent of local changes in the viewed text. They suggested that reading patterns employed "buffers" within memory. A buffer was a memory device used to store incoming information and keep it available for later stages of processing. The theory held that incoming information was accumulated and stored in buffers so that processing could continue after the eyes moved on to another fixation. The eyes would fixate at a relatively constant rate, say 200 milliseconds. If the amount of incoming information was 250 milliseconds long, some of the processing of that longer fixation would be carried over into the next fixation. The processing would "catch up" on some future fixation which took less that the constant 200 millisecond fixation time. For simple texts, this lagging and catching up process would not accumulate.

For difficult text passages, the buffer reached its storage limit. The result of the full buffer was a shift to a slower processing rate with longer fixation durations so the system could complete the processing of new information before the eyes moved on. Overall text difficulty level was the determining factor for average fixation duration. Difficulty with local areas of a text was not reflected in individual fixation durations. The overall rate remained relatively stable depending on the global difficulty level.

Potter (1983) presented evidence for eight separate buffers operating within eye fixations. These buffers performed specific input, storage, and output tasks controlling factors such as spatial visual memory, conceptual short term memory, working memory, and location and timing of the next saccade (eye jump). Incoming information within one fixation was stored in a buffer while one type of processing occurred and then transferred to another type of buffer for a different type of processing. This storage, processing, transferring sequence occurred several times for the information acquired during each fixation. The buffers permitted a "decoupling" of the eye and mind thus making the relationship between eye movements and mental processing less direct.

The other theory about text properties in relation to eye fixations was called the "local control" theory. Just and Carpenter (1980) outlined the main elements of this theory. The immediacy assumption of this theory suggested that the reader tried to interpret the meaning of each word immediately upon encountering it in the text. Interpretation of words was not delayed waiting for a group of words to accumulate before the interpretation was done. Another important aspect of the theory was the eye-mind assumption. The eye-mind assumption stated that a reader continued to fixate on a word until some level of cognitive processing had been achieved based on some criterion. This continued processing was measured as the gaze duration of a given word. Processing of the currently fixated word did not mean only that word could be processed since concepts from previously fixated words were available for use from memory.

Evidence for local control of eye movement patterns is substantial. Fixation durations were found to be effected by local text properties. O'Regan (1979, 1980, & 1981) found that fixation durations were longer on shorter words, and when incorrect letters were in the peripheral area on a previous fixation. Fixation durations were shorter when the fixation was at the beginning or end of a word rather than in the middle. Rayner (1975, 1977) found that the duration of the first fixation in a line of text was longer and the last fixation in a line was shorter. Fixation durations on the areas between sentences were found to be shorter and fixations on low frequency words were found to be longer. Kliegl, Olson, and Davidson (1983) found fixation durations were longer on low frequency words and when there was only one fixation on a word. Fixation duration was influenced by the length and frequency of words which fell outside the direct fixation areas. Just and Carpenter (1980) found that fixation durations increase for infrequent words and decreased for modified nouns whose referent was easily inferred.

Kennedy and Murray (1987) compared good readers to poor readers when reading sentences of two difficulty levels. Good readers controlled total reading time by increasing the total number of fixations and not changing their average fixation duration. Poor readers showed no change in the number of fixations for the two types of sentences, but had somewhat longer fixations on the more difficult sentences. Poor readers made more fixations than good readers when reading sentences containing questions.

Shebilske and Fisher (1981) found fixation durations to be longer in areas of the text containing more important ideas than in other areas. Underwood, Hyona, and Niemi (1987) studied information zones within words; they found that these important zones effected fixation durations. More important zones were fixated longer. Frequency of fixations was higher in important information zones within words.

The consensus of these studies was that local properties of the viewed text directly influenced temporal eye movement patterns. Fixation durations were longer or shorter according to factors in the local areas of the text like word frequency, length, or meaning.

The issue of global versus local control of eye movements created a third issue. Did evidence of local control of eye movements imply immediate control? Immediate control of eye movements meant that the fixation duration and following eye jump were controlled by information from the current fixation and not from information obtained in previous fixations. Evidence of local control did not necessarily mean immediate control.

Support for this view that the current fixation controlled eye movements came from Underwood and McConkie (1981), Rayner (1975), Rayner and Pollatsek (1981), O'Regan (1981), McConkie, Zola, and Wolverton (1980), and Underwood, Hyona, and Niemi (1987). These researchers used various stimulus presentation techniques to control the amount of information presented in the foveal and peripheral viewing regions. By controlling the amount of information presented, conclusions about the exact time at which certain pieces of information were processed could be made. Results indicated that the current fixation had direct effects on eye movement patterns. The eye guidance system was controlled from moment to moment by properties of the text. Fixation duration and frequency, and fixation location were effected by information from current fixations.

The importance of the local control theory was in the relationship between eye fixation patterns and cognitive processing. If a direct link between local text difficulties and fixation time were established, then fixation patterns were an immediate measure of cognitive processing difficulty. If the global theory were found more appropriate, then this eye-mind link would be less direct. Bouma (1978), Bouma and deVoogd (1974), Kolers (1976), and Shebilske (1975) supported the view that there was insufficient time during fixation for immediate control mechanisms to operate. Buffering, transferring, and interpreting incoming information took up more time than the average of 200 to 250 milliseconds per fixation. Just and Carpenter (1980) and McConkie (1979), on the other hand, relied on the immediacy assumption to establish the validity of their theories. Moment to moment processing was a direct measure of cognitive processing. These issues between global, local, and immediate control of eye movements have yet to be resolved.

2.4 Viewing Pictures and Natural Scenes

Pictures and natural scenes (referred to as scenes) can be thought of a "natural" information as opposed to text which is "artificial" information. This distinction refers to the elements of the viewing field which carry the encoded information. Text is artificial in the sense that meaning is encoded using a set of constructed symbols placed in a pattern. Scenes are natural in the sense that they are made up of representations or caricatures of real world objects.

The elements of a scene are not in the same predictable, constrained order as a text. Text is constrained by the sequence of letters in words, the sequence of words in a sentence, and the sequence of sentences in a paragraph. Scenes use elements which are of varying size, shape, position, and relationship in the visual field. Scenes are constrained in the sense that certain relationships are expected. For example, the sky is up and the earth is down, or birds fly and horses walk. But, these constraints are much less rigid than those in texts.

The same spatial and temporal characteristics of eye movement patterns studied in reading research are important to viewing scenes. Spatial characteristics of fixation location and sequence in scenes are viewed in a different way than in texts. In scenes, the location and sequence of fixations is much less constrained by the elements of the information array. Normal English text is written in a left to right pattern of lines from the top to the bottom of a page. Processing of text requires a general left-right-return pattern, with some variations for regressive eye movements.

Scene processing does not rely on an external pattern for basic control of eye movements. Questions of fixation location and sequence become ones of predictability. Given the present location, what is the probability that the next fixation will be located in a given area? Or, is there a predictable, stable pattern used when viewing scenes? The control of the spatial patterns of viewing scenes becomes more important than for texts because of the lack of structure inherent in scenes.

Fixation sequences in viewing scenes were termed "scanpaths." Norton and Stark (1971) originally formulated the scanpaths theory postulating a repetitious, sequential, scanning pattern controlled cognitively by a subject. Gale and Worthington (1983) studied the effects of training subjects to use a scanning strategy to direct their scanpath when viewing chest X-rays, finding some negative effects of the training. Nodine and Kundel (1987) studied viewing strategies when searching chest X-rays for tumors; global search patterns produced less effective screening than local intense viewing in known target areas. Fisher, Karsh, Breitenbach, and Barnette (1983) investigated scanpaths in relation to recognition tasks involving the repeated viewing of identical or similar pictures, concluding that scanpaths were loosely controlled by the pattern of information areas in the viewing scene. Stark and Ellis (1981) refined the research of Norton and Stark by using more realistic pictures and ambiguous figures to chart scanpaths; they concluded scanning patterns reflect changes in the cognitive states of the subject. Antes and Pentland (1981) studied the effects on scanpaths of the presence of unexpected objects in the viewing area. They found that within subjects there appears to be a pattern strategy employed when viewing low context pictures, but this was not found to hold between subjects. The results of the research on scanpaths was not unequivocal concerning either their existence or their importance in visual and cognitive processing.

Another viewing pattern studied was perceptual scan, or the size of the useful field of view in a fixation. Studies by McConkie & Rayner (1975) and Rayner (1975) found that when processing text, the only text analyzed for semantic content was in an area within +/- one degree of angular rotation from the center of the fixation. This is generally given to be the size of the foveal area of the eye. Other information about the text is obtained from the peripheral area up to about +/- four degrees of the current point of fixation, but this information is about word length and placement, not meaning. Nelson and Loftus (1980) studied perceptual scan in relation to pictures. They concluded that when viewing pictures, the eye acquired useful, substantive information from an area of +/- two degrees of angular rotation.

Loftus (1972, 1981) studied fixation duration in relation to viewing pictures. Originally he hypothesized that memory performance increased when a large number of short fixations were made on a scene. Later research under tighter experimental control revealed that fixation duration was critical to performance on a memory task. As fixation duration increased, so did performance on the memory task.

Loftus and Mackworth (1978), Goodman and Loftus (1981), and Loftus (1981) studied the effect of important information areas in a scene. These were areas which contained information which differentiated the viewed scene from others of the same type, or information which was unexpected or informative. The conclusion was that these important areas were fixated earlier, more often, and with longer fixation durations.

Locher and Nodine (1987) studied the effect of symmetry in viewing abstract art. They differentiated between survey fixations (lengths of 100-300 milliseconds) and examination fixations (length greater than 400 milliseconds). They found 65% of fixations were of the survey type, while only 18% were examination fixations. When single or double symmetry was present in a complex display, the axes of symmetry attracted the eye such that fixations were concentrated along the axes of symmetry during an exploratory viewing. However, the number of survey and examination fixations were not influenced by the presence or absence of symmetry.

Molnar and Ratsikas (1987) studied the effect of aesthetic motivation on viewing patterns. They told one group of subjects that questions about the content of art works would be asked after viewing. The other group of subjects was told they would be asked questions about aesthetics of the art works viewed. Average fixation durations were significantly higher for the aesthetic group. There was no difference between the groups for the spatial issue of saccade length.

Viewing scenes and reading texts have important similarities and differences. Fixation sequences are more predictable when reading texts, but the presence of important or unexpected information in a scene greatly increases the predictability of fixation sequences. The effective perceptual span of a fixation is about twice as wide for scenes as for test (+/- four degrees versus +/- two degrees). Fixation duration in viewing scenes is related to the characteristics of the elements viewed and longer fixations are associated with concentrated inspection of important or unexpected elements of the scene, as with text reading. Symmetry and aesthetic properties do not refer to text, but have effects on eye movement patterns when viewing scenes.

2.5 Graphical Interpretation Theories

Graphical interpretation theories attempt to explain the cognitive process of decoding information from a graph. More specifically, these theories attempt to explain how the human visual system is able to perceive the symbolic elements of a graph and their interrelationships and how the brain translates this information into a meaningful structure. The basis of these theories is visual information processing theory.

2.6 Visual Information Processing

Visual information processing theory (Gaarder, 1975) uses the concept of biofeedback to explain how the brain mediates the process of visual perception. This feedback model holds that there is an interconnection between the eye and the brain which controls what information is transmitted to the brain for processing. The eye is not just a receptor, like a camera, which simply records any and all information it receives. Feedback, prompted by individual bits of information to the brain, is sent to the eye to regulate the next bit of information to be sent. The amount of information received by the eye is much more than the amount sent on to the brain. The feedback process acts like a filter selecting only some information for transmission to the brain (see Figure 2.)

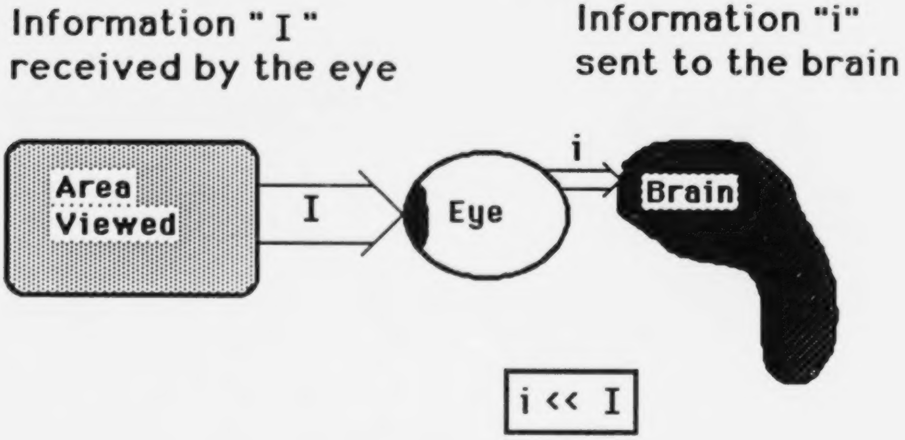

Information "I" received by the eye
Information "i" sent to the brain
Area Viewed
I
Eye
i
Brain
i << I

Figure 2: Biofeedback model of visual perception.

Information reaching the brain is first screened for visual patterns of lines, colors, textures, or large continuous areas. This pre-processing produces a "visual sketch" (Kosslyn, 1985). The first phase of processing involves discrimination of large or obvious differences in the visual information. Julesz (1981) refers to this as a preattentive visual process which instantaneously perceives elements of the visual field with little mental effort. These discriminations are organized into perceptual units which are then processed in the next phase. For example, four equal lines that enclose an area are seen as a square instead of separate lines.

These perceptual units are then stored in short-term, or working memory. At this stage, information critical for correct interpretation of the perceptual units is accessed from long-term memory. General background knowledge and specific knowledge is associated with the perceptual units. If no mental frame of reference for the perceptual unit exists, then a conscious reorganization of the viewed pattern in short term memory leads to a new attempt to interpret the pattern using stored information (Kosslyn and Pinker, 1983). Only when the proper information is referenced from long-term memory is the interpretation process completed.

The "frames" theory of cognitive behavior helps explain the mental processing involved in acquiring meaning from the perceptual units. The cognitive theory of frames, as originally proposed by Minsky (1975) and refined by Davis (1980), provides a mental structure which helps explain cognitive decoding of incoming information. A mental frame is defined to be "...a specific information-representation structure that a person can build up in his or her memory and can subsequently retrieve from memory when it is needed" (Davis, 1980, p. 170). Frames serve as assimilation schemas for organizing incoming information. If the incoming information is not complete, a frame is able to provide default values for critical areas so that the frame may be used effectively. Frames are persistent over time allowing the individual to operate in the same pattern regardless of the incoming information.

2.7 Graphical Perception Tasks

Cleveland and McGill (1985) proposed a theory of graphical perception to explain how quantitative information was extracted from data graphs. Graphical perception was broken down into separate elementary decoding tasks. Visual decoding for these tasks was defined as "...the instantaneous perception of the visual field that comes without apparent mental effort" (Cleveland and McGill, 1985). This instantaneous perception was what Julesz (1981) called preattentive vision.

Cognitive decoding of graphs was separate from graphical perception. Cognitive decoding of information in a graph, such as scale or slope, was similar to decoding of other types of quantitative information such as a table of numbers. The power of graphs came from the ability of the preattentive visual system to recognize geometric shapes and patterns and judge size relationships.

Ten elementary perceptual tasks were identified and ordered relative to the ease of decoding. The higher an elementary perceptual task ranked on the list, the less the expected error when that perceptual task was the main discriminatory factor in interpreting a data graph. (The tasks and their ordering is shown in Figure 3.)

Rank	Aspect Judged
1	Position along a common scale
2	Position on identical but nonaligned scales
3	Length
4	Angle, Slope
5	Area
6	Volume, Density, Color saturation
7	Color hue

Figure 3: Rank ordering of graph interpretation tasks (Cleveland and McGill, 1985)

Cartesian graphs were understandable because visual information is decoded by perceiving position along a common scale. This decoding by common scale occurred for both the horizontal and vertical axes. But the real power of a cartesian graph came from the ability to perceive the horizontal and vertical values simultaneously and not separately. The relationship of points (x_i, y_i) and (x_j, y_j) in the cartesian plane was the slope of the line segment joining the two points. The visual system easily detected the slope relationship in a graph by imagining a smooth curve through the points (Cleveland and McGill, 1984).

For example, Figure 4 contains a cartesian graph of data points. Slope judgements between individual points and for multiple points can be made. Slope between the points A and B is judged to be less than one, while slope between points C and D is much greater than one. General slope judgements indicate that the slope greater on the left side of the graph than on the right. The eye-mind system judges the relationship in the graph to be non-linear.

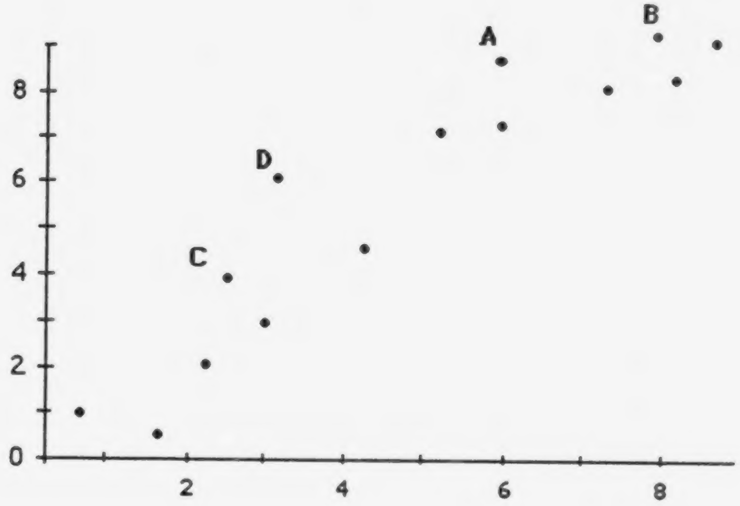


Figure 4: A cartesian graph of data points.

Compare Figure 4 with the bar graph in Figure 5. The information in both graphs is the same, except the slope information has been removed from the graph in Figure 5. The x and y values can still be judged by position along a common scale. Slope determination for pairs of individual points and for overall trends can not be perceived as easily, if at all.

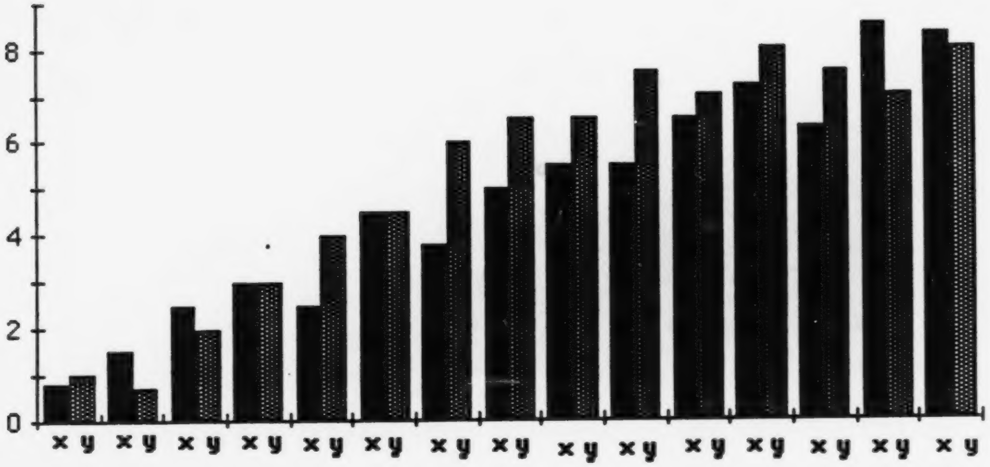


Figure 5: A bar graph of the same paired x and y values expressed in Figure 4.

This theory of graphical perception was developed using paper and pencil performance tasks. No eye movement data were collected to substantiate the claims of the theory, yet statements about eye movement patterns are made in relation to decoding of the elementary graphical perception tasks. Referring to an example of a cartesian data graph showing the divorce rate over the last 50 years, Cleveland stated:

> Once the variables being graphed are understood, we can extract quantitative information..... at a very elementary mental-visual level. We derive this information by scanning the plotting symbols, the connecting lines, and the scale lines, and without consciously looking at the tick mark labels (Cleveland, 1985, p. 230).

Direct evidence on eye movement patterns in relation to these elementary graphical perception tasks would help researchers understand each of the tasks and the interrelationships between them during graphical decoding.

2.8 Graph Comprehension

Pinker (1981) proposed a theory of graph comprehension which was broader in scope than Cleveland And McGill's theory. Pinker's theory addressed cognitive graph comprehension rather than just preattentive visual processing. The theory made three claims. First, a large number of two-dimensional shape attributes were easily and quickly identified by the human visual system. These attributes included length, shape, orientation, height, smoothness, continuity, curvature, parallelism, density, and others. Second, various aspects of data sets were translated into different visual patterns. For example, some data sets translated as parallel lines, some as intersecting lines. Third, experienced graphs readers knew the relationship between quantitative trends and visual patterns for different types of graphs. When decoding a graph, experienced graph readers looked for higher order visual patterns such as overall shape or trend without having to refer to individual points to compare them one by one.

Predictions of the degree of difficulty a graph reader had when attempting to decode certain types of information from certain types of graphs was summarized in one statement:

> The ease of reading a certain type of information from a certain graph format will depend on the extent to which that graph format translates that trend into a single visual pattern that the visual system can automatically extract, and on the extent to which the reader knows that the correspondence in that format between the quantitative trend and the visual pattern holds (Pinker, 1983, p. 5).

The power of a graph to transmit information was not uniform across the types of data presented and questions to be answered. One type of graph could be good or poor at answering questions because of the way geometric patterns were encoded in the graph and the experience of the graph reader.

Pinker (1983) conducted three experiments to test this theory. Results indicated that the length of elements in the graph was easier to decode than angle relationships when reading individual values of the graph. But, when dealing with global trends, angle relationships between the elements were easier to decode than length of elements. Subjects were able to translate visual patterns of the graphs directly to global trends without looking at individual points when they were familiar with the type of graph being viewed. Even when not given explicit instructions about the trend-shape correspondence in a graph, subjects were able to interpret trends effectively when the encoding variable (ie. length or angle) was easy to decode. Graphs were easy or difficult to decode depending on the type of information to be extracted and the naturally perceivable visual patterns.

Pinker used paper and pencil performance tasks to gather data in his experiments. No eye movement data were gathered to substantiate his findings on why different types of graphs were easy or difficult to decode. The question of whether subjects really translated visual patterns directly to global trends without looking at individual points of the graph could have been answered using eye movement data.

2.9 "How To" Literature

Tufte's (1983) book is an example of a "how to" treatment of graphical interpretation. Several works of this nature (Chambers, Cleveland, Kleimer and Tukey, 1983; Cleveland, 1985; Fisher, 1982; Schmidt, 1983) are available. They treat the topics of statistical data graphs and maps. Their purpose is to establish principles of the effective construction and use of data graphs and charts.

2.10 Summary

Research using eye movement data on reading text and viewing pictures and natural scenes demonstrates the importance of variables such as fixation duration, fixation frequency, and fixation location. Texts and scenes are not processed in the same manner. The nature of graphs is a combination of the nature of texts and scenes. How these important variables effect graph reading and comprehension is not known.

Graph interpretation theories provide insights into the power of the graphical symbol system to convey meaning. Identification of the elements of graphical perception allows investigation of the effects of each individual element in the overall process of graph reading. Knowing what makes a graph easy or difficult to understand can clarify how information is processed cognitively.

CHAPTER III

METHODS AND PROCEDURES

3.1 Introduction

The research study described in this chapter took place during Autumn Quarter, 1986 and Winter Quarter, 1987 at The Ohio State University, Columbus, Ohio. The purpose of this research study was to define and investigate the differences between novice and expert graph readers on the dimensions of average fixation duration, percent of total viewing time, and performance on a selection and memory task. A further consideration is to relate the findings on viewing time factors to the design of new and more effective graphing skills curriculum for middle and high schools. This chapter contains a description of the equipment, populations, procedures, the instrument, the analysis techniques, the limitations of the study, and the statistical analyses used.

3.2 Equipment Used in the Study

Data on eye fixation location and duration were gathered using a MicroMeasurements System 1200 Eye Monitor. This instrument uses an RCA TC2511/U infrared sensitive television camera to track the pupil of a viewer's eye. The system has a range of +/- 20 degrees with a sampling rate of 60 Hz. An IBM

PC/XT micro computer with an IBM Data Acquisition and Control board was used to present graphical displays, collect and organize the data, and display it in several appropriate formats.

The IBM Data Acquisition and Control board is a digital/analog conversion device installed in the IBM micro computer. This device allowed the collection or transmission of analog (continuous) or digital (discrete) data and the conversion of data from one form to the other. Operation of the Data Acquisition and Control board is through machine language subroutines accessed from the operating programs. Data is fed into the IBM computer from the eye tracking instrument through the Data Acquisition and Control board and stored in a numerical array within the operating program.

3.3 Definitions

Digital data coming from the eye tracking instrument is in the form of a 16 bit binary number which is updated 60 times a second. The upper 8 bits represent the vertical position of the eye, and the lower 8 bits represent the horizontal position. A single data point is one 16 bit binary number. Each single data point is made up of a location in the viewing field and a set duration of 1/60 th of a second. The location of a data point is given as a set of coordinates in the cartesian plane with the origin (point (0,0)) in the middle of the viewing field. When the eye is looking to the upper right of the field, both coordinates of each data point are positive. When looking to the lower right, the coordinates of each data point have a positive horizontal value and a negative vertical value. The viewing field is organized in exactly the same manner as mathematical graphs in a four quadrant cartesian plane.

Fixation duration is the length of time the eye fixates on a specific location in the viewing field. Duration of a fixation is calculated by multiplying the number of single data points at a specific location by 1/60. For example, is a single fixation was made up of 25 single data points, then the fixation duration would be .417 seconds or 417 milliseconds.

A minimum fixation is defined to be a fixation in one location for at least 50 milliseconds (.05 seconds). At the rate of 60 data points per second, 3 data points represent exactly 50 milliseconds, the length of a minimum fixation. In order to constitute a minimum fixation, the horizontal and vertical position of the eye must be the same for three successive data points. This sampling rate is three times faster than a minimum fixation, and is sensitive enough to differentiate accurately between actual fixations and data gathered during eye jumps.

The neighborhood of a fixation is an area within +/- 2 measurement units in the horizontal and vertical direction of a specific location. For example, if the location of a fixation registered as (-15, +12) then the neighborhood of that fixation would be between -17 and -13 in the horizontal direction and between +10 and +14 in the vertical direction. Because of the presence of microscopic eye tremors and the sensitivity of the MicroMeasurements 1200 machine, the location of a fixation may not register exactly the same x-y coordinate reference throughout the entire fixation. Establishing a neighborhood of proximity around a fixation allows the correct measurement of the duration and location of the fixation.

The process of calculating the duration of the fixation has several steps. After the beginning of a fixation is established by the minimum fixation definition of three data points, each successive data point is examined to see if it falls within the defined neighborhood of the established minimum fixation. Data points that are in the defined neighborhood are added to the current fixation. The end of the current fixation is signaled when a single data point fails the neighborhood test of +/- 2 measurement units. Once a fixation is terminated, its length in seconds is determined by the number of data points in the neighborhood multiplied by 1/60, the length in seconds of one data point. The location of the fixation is taken to be the average of the coordinate values of the points in the fixation for both the horizontal and vertical directions.

A sustained fixation is a fixation of 50 msec. or more within a defined neighborhood of the fixation without moving out of that neighborhood. Movement out of a neighborhood and then back into the same neighborhood is defined to be two separate fixations in that neighborhood. Data smoothing involves combining all data points which occur in a given neighborhood of a fixation into one fixation with a duration and a location.

3.4 Populations

Two populations were operationally defined for the purposes of the study. The "expert" population was defined to be graduate students and professors in mathematics and mathematics education currently enrolled or employed at The Ohio State University. The underlying qualification for a subject's inclusion in the expert group was the amount of experience he/she had in reading, constructing, and interpreting mathematical line graphs. A significant part of this experience with mathematical graphs comes from the process of learning calculus and other higher level mathematical subjects. It is in these courses that graphs are used to display relationships, develop concepts, and solve problems.

The "novice" population was defined to be students who placed in level 4 or 5 on the OSU Mathematics Placement Test and who were currently enrolled in Math 050 or Math 075 at The Ohio State University. The OSU Mathematics Placement Test is the test given to all entering students to determine their appropriate placement level in the mathematics course sequence. A placement in level 4 or 5 on this test means that these students are considered "remedial" and must take non-credit courses to make up their deficiencies before credit courses in mathematics can be taken. Math 050 and Math 075 are the two non-credit remedial courses given at The Ohio State University. They consist of a review of high school algebra.

The two populations chosen were both from groups at The Ohio State University for two reasons. First, the availability of qualified subjects is much greater in the university community. Subjects considered expert in reading and using mathematical graphs would be difficult to identify in a general population. The availability of novice subjects at Ohio State is also great because of the large number of students who qualify for remedial mathematics.

The other reason for the selection of the groups of subjects from the University community had to do with age and development. No matter how the novice population was defined, the expert population had to be defined much as it was. Choosing a group of novice subjects who where in middle school, on the other hand, would have introduced unavoidable developmental differences between the two groups studied. In an effort to reduce the error and more carefully measure the actual graphical processing differences between experts and novices, the two groups needed to be closer in age.

During the pilot study, an expert/novice dichotomy was studied. In that study, experts were from the same population as defined for this study. The novice group in the pilot study was selected from graduate students at the University who were classified as nonmathematical because of their background and training. The varied backgrounds of this novice group actually caused some concern because of the wide differences from subject to subject. There was little or no control over whether these subjects really had much experience with using mathematical graphs. It was decided that the novice group for the larger research study should come from a more homogeneous population. Hence, the novices were defined as stated above.

Novice subjects were chosen on a volunteer basis from random Math 050 and Math 075 classes at the University. Expert subjects were also volunteers, but from a much smaller base population than the novice group. Expert subjects included in this study did not participate in the pilot study. The original research plan was for 25 subjects in each group. New subjects were added to each group until 25 valid data sets for each group were collected. In all, data was collected from 28 novice subjects and 27 expert subjects. Three novices and two experts were eliminated from the study because of collection problems with the IBM computer and problems with certain properties of eyes. Early in the data collection process, three subjects were lost from the study because of disk errors in saving data. One subject was eliminated because she was wearing dark eye liner and the eye tracking equipment would not track her pupil. Another subject's data was invalid because he did not open his eyes far enough for the eye tracker to get a good view of the pupil. The total number of subjects whose data was subjected to analysis was 50, 25 experts and 25 novices.

3.5 Instrumentation

Subjects in the study viewed six mathematical graphs. All graphs were similar enough so that they contained the same physical features. The graphs were of continuous polynomial functions having features such as maximum and minimum values, axis intercepts, and smooth, continuous shape. The graphs represented quadratic and cubic polynomial functions in a four quadrant plane (see Appendix A, Figures 14 through 19). During the presentation to the subjects, the graphs were generated on an IBM PC computer in high resolution graphics mode (640 x 200 resolution). Each of the graphs was plotted on axes that were scaled identically.

The performance task consisted of a set of five multiple choice distractors for each of the six graphs (see Appendix A, Figures 20 through 25). The distractors were created to be similar in shape to the actual graph, but differing in important aspects such as intercept values, maximum and minimum points, or spread of the function. Sorting for the correct distractor depended on the subject remembering important numerical information about the viewed graph. The distractors for each graph were presented on one sheet of paper and were large enough so that information about the viewed graph could be written directly on the chosen distractor.

3.6 Procedures of the Study

Eye tracking data were collected on an individual basis with one member of the population being observed at a time. The order in which the subjects were tested was not important since the data collection was on an individual basis. Data were gathered on fixation duration and the horizontal and vertical position of the eye in the visual display. Raw data were stored directly on magnetic computer disks and could be repeatedly analyzed in exactly the same configuration as when the original tests were performed.

The data collection procedure consisted of 8 trials with the first and last trial being a scaling routine for calibrating the eye tracking machine to the IBM computer. The scaling routine consisted of reading an array of digits (1 to 9) in a three rows by three columns pattern (see Appendix A, Figure 26). Because of individual physiological differences in the eye, each subject had a different scale. Two scaling trials were done for more correct calculation of the scale for each subject.

Each of the six graph reading trials was a unit in that a graph was read and the accompanying task was performed before another graph and task were presented. An effort was made to minimize confusion factors from one graph to the next by making each trial separate. Data storage time allowed subjects time between trials to relax and prepare for the next trial.

The second trial in the data collection procedure was a practice session using graph #1 (see Appendix A, Figure 14). All subjects used the same practice graph. All procedures were the same during the practice graph reading as during the actual treatment except no data were stored on disk from the practice graph. Subjects were aware that the first graph reading was a practice session. Subjects were able to get a feel for the viewing and performance tasks by doing this practice graph. After the practice task, subjects were asked if there were any questions or concerns about what they were expected to do. Of all the subjects tested, only two had any questions after the practice session.

The other five graph reading trials were for data collection. The five graphs presented in these five trials were the same for each subject. The order of presentation of the five graphs was rotated. Every sixth subject viewed the graphs in the same order. This helped spread the training effect during the data collection sessions evenly across all the trials.

Subjects were given general instructions before the viewing sessions began. The instructions included information about the order of the trials in the sequence and the nature and purpose of the scaling trials. They were told that they would have a practice trial where no data would be collected. They were not given information about the specific mathematical functions represented or other clues about the graphs except that the graphs were mathematical graphs.

Instructions about the performance task were general in nature. Subjects were shown the set of distractors for the practice graph as an example of what the task work sheets looked like. They were told to find among the five choices the graph they had just viewed and to write down important information about that graph on the chosen distractor. Subjects were told that they could write down any information they thought was important including numbers, words, descriptions, or other things they chose. Subjects were not instructed to include any specific pieces of information and the words "intercept," "scale," "maximum," and "minimum" were not use in the instructions. When doing the performance task, subjects were allowed as much time as they wished.

Graphs were viewed for up to one minute each. Subjects were instructed to view each graph for as long as they wished up to the one minute limit. They were told that speed was not a factor in their performance. Each viewing session began with a signal from the operator to begin reading the graph. Subjects immediately moved a switch to activate the presentation monitor and the graph appeared. When the subject chose to end the viewing, he/she moved a switch to turn off the monitor showing the graph. The elapsed time for reading the graph was recorded using a stopwatch. (If the one minute viewing limit was reached, the graph disappeared from the monitor.) Immediately after turning off the monitor, the subjects were given the performance task. Subjects were allowed to remove their heads from the head rest and fill out the performance task work sheet.

3.7 Analysis Techniques

Analysis of the eye scanning data on average fixation duration and percent of total viewing time was done by superimposing a square grid system over each graph. The graphs were blocked with a grid of .833 inch squares which represents half the unit length for the chosen scale of the graphs. The grid was positioned in such a manner that the origin of the coordinate system is in the center of a grid square (see Figure 6). There were 9 rows and 11 columns making 99 separate areas or blocks in each graph. The blocks were all of equal area, so the probability of a random fixation in any one square was equal to approximately .01 or 1%. Specific features of the graphs, such as scale values or axis intercepts, were blocked in the viewing area. Measurement of the two dependent variables was per block. This means for example, that when an average fixation duration was reported, it was the average fixation for those fixations which occurred within the limits of a particular block. When the percent of total time was calculated, it was the percent of the total time that was spent in the different blocks of the graphs.

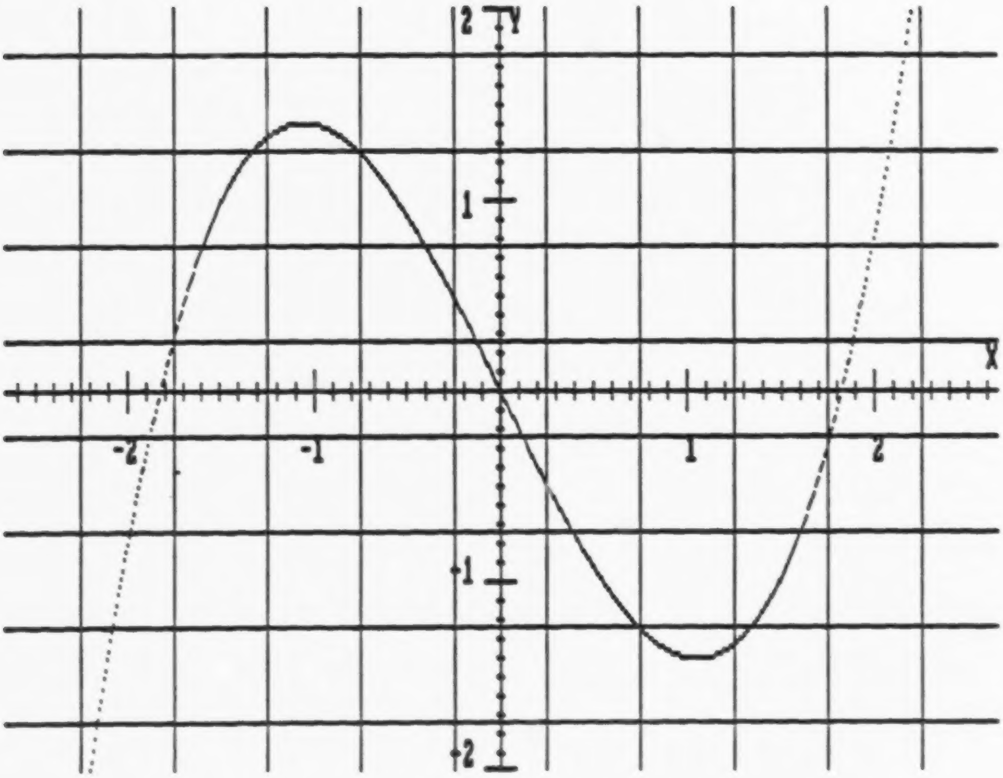


Figure 6: Grid analysis system superimposed over graph #4.

This blocking technique allowed selection of specific regions for the information contained within them. For example, blocks containing axes intercepts were isolated for investigation. In the same way that a linguist or reading researcher identifies key function words or parts of speech within a text passage, blocking a graph allowed identification of places in the viewing area that contained important information for the correct interpretation of a graph. The determination of the importance of certain pieces of information in the graph is well-established by the function of these pieces of information in the graph and by mathematical convention.

Blocks of the graphs were labeled in numerical order starting from the upper left corner from 1 and proceeding to the right to 11. The second row contained blocks number 12 through 22, and so on. The numbering scheme was for differentiation, not to indicate any ranking of importance.

Total viewing time in a given block was the sum of all fixations within that area. Viewing time for each graph read was different for each subject. Total viewing time in a grid area was not reported in seconds, but as a percent of the total elapsed time. Each individual graph reader spent varying elapsed time viewing each graph because of individual differences. For an effective comparison, the elapsed time factor was equalized across subjects by using the percent of elapsed time data.

Maps showing the dependent variables for each block of each graph were created for each subject. The individual subject maps were the basis of comparison for the expert and novice groups of subjects. For the five graphs in the study, there were two maps for each of the 50 subjects for a total of 500 maps. For each subject, one map showed the percent viewing time and another showed the average fixation duration over each block in the graph. (see Appendix A, Figures 27 and 28).

Data from the performance task was in the form of a score between 0 and 12 on each of five graphs, or a score between 0 and

60 for the aggregate of the five graphs. Points on the performance task were awarded on the basis of the subjects' choices of graphs from the sets of distractors and the accuracy of the additional information they provided. If a subject did not choose the correct distractor for the viewed graph, no points were awarded even if the additional numerical information provided was correct. For each graph, two points were awarded for each of the following categories:

1. correct graph
2. x intercepts
3. y intercepts
4. correct scale in the horizontal and vertical direction
5. relative or absolute maximum values
6. relative or absolute minimum values

3.8 Statistical Design

The statistical design of the study was a multivariate factorial design comparing experts to novices across selected blocks of the five graphs. Blocks of a graph selected for analysis were determined by their content. Three categories of blocks were established before analysis began. They are:

1. important blocks
2. less important blocks
3. unimportant blocks

Important blocks are defined to be those blocks of a graph which contain essential information concerning the correct interpretation of the graph. These essential pieces of information include the origin (point (0,0), x-axis intercepts, y-axis intercepts, relative maximum values, relative minimum values, and scale values directly related to maximum and minimum values.

Less important blocks are defined to be those blocks of a graph which contain pieces of information about the graph other than the important blocks. These less important pieces of information include general scale values, shape of the curve, and labels.

Unimportant blocks are defined to be those blocks of a graph containing no information about the graph or blocks where both groups of subjects spent less than or equal to 1% of their total time. The 1% level represents what might be expected for a completely random distribution of fixations on the 99 blocks. A majority of the 99 blocks of each graph fall into this category.

Analysis of the eye tracking data was done using Multivariate Analysis of Variance (MANOVA) from the SPSS-X statistical analysis computer program. Important blocks of a graph were identified. These blocks were then analyzed for differences between experts and novices for the percent of total time and average fixation duration variables. The same procedure was followed for less important blocks of the graph. These procedures were repeated for each of the five graphs in the study.

Further statistical analysis of the eye tracking data was done using two different correlation coefficients. The magnitudes of the cell means for the two dependent variables over each group were ranked. Spearman Rank Correlation Coefficients were calculated for comparisons between and within experts and novices for the two dependent variable. Pearson Product-Moment Correlation Coefficients were calculated for the same comparisons using the cell means.

Statistical analysis of the data from the performance task was a t-test of the difference between the two group means.

3.9 Limitations

There are two important limitations of this study which must be addressed. One concerns the statistical analysis and the other the task performed by the subjects in the study.

Data collection using eye tracking equipment is time consuming and expensive because it is done on an individual basis. Also, the quantity of data from one subject for one graph is large. One minute of data constitutes 3600 pairs of data values. Each subject had up to 5 minutes of data. As a result, the size of the sample is relatively small. This means that when using the MANOVA statistical procedure, care must be taken to limit the number of variables introduced into the matrix at one time. Entering all 99 blocks of a graph into the analysis at one time would cause erroneous results in the analysis. Hence, the number of blocks entered into the analysis at any one time must be limited to less than 12, with a better value being 7 or 8.

The task of choosing the graph just viewed and entering values for specific points from memory had some limitations. The task was a memory task. More importantly, the task was intended to focus the subjects' attention on what they considered to be the most important aspects of the graphs. Asking one or more specific questions about one graph would have biased the subjects' focus on the following graph. For example, if subjects were asked to identify the y-intercept after viewing the first graph, then they would probably attend to the y-intercept first or more often on the second graph. On the other hand, too general a task would not have focused the subjects' attention in any manner. To say "Just look at these graphs" was a non-task. This study's purpose was to identify important variables related to graph interpretation. This performance task made subjects focus on what they considered important when reading a mathematical graph. Evidence from a pilot study supported this belief.

CHAPTER IV

STATISTICAL ANALYSIS

4.1 Introduction

Statistical analysis of the eye tracking data from the five graphs used in this study was separated into two general categories: 1) important blocks of the graphs, and 2) less important blocks of the graphs. Important blocks were the areas of the graphs which contained important information for interpretation of the graph. The less important blocks were the areas of the graphs which contained information which was not as critical as the important blocks for the interpretation. The two-way factorial design of the study produced two main effect terms and one interaction term between the main effects. The two main effects were labeled "block" and "training." The "block" independent variable referred to the areas or blocks of the graphs used as the units of analysis. The "training" independent variable referred to the expert versus novice levels of training.

The two dependent variables defined in the study were: 1) the percent of total time spent within blocks of the graphs, and 2) the average fixation duration within blocks of the graphs. Multivariate analysis of variance (MANOVA) was the statistical procedure used to test for differences between the groups of subjects. The MANOVA procedure produced univariate results to test the two dependent variables separately within the two main effects.

The main purpose of this study was to explore the differences between experts and novices when viewing mathematical graphs. In the statistical analysis, the "training" independent variable defined this difference. Since there were only two levels of training, post hoc analyses procedures were not needed to delineate significant differences between the levels of training. In all cases where significant differences were found, the experts had higher mean values than the novices.

The main effect for block did not test for differences between experts and novices. The block main effect pooled all subjects into one group and tested for differences between blocks for the two dependent variables. For example, significant main effects for block indicated that when different blocks of a graph were viewed, subjects (as one group) had significant differences from block to block for the two dependent variables.

Univariate significance for the two dependent variables under the block main effect had a similar interpretation. For example, if significant differences were indicated for average fixation duration under the main effect for block, then the conclusion was that subjects as a group had different average fixation durations when looking at different blocks of a graph.

Results of the main effect for block did not produce important information about the differences between experts and novices. Information from the analysis of this main effect gave insight into the graphical symbol system itself. The lack of significant interaction between training and block was indicative of the parallelism in the data across blocks of the graph. This parallelism also gave insight into the power of the symbol system. The implications about the symbol system will be discussed in the next chapter.

Information about the differences between experts and novices in how they attended to the blocks of the graphs was obtained through two correlations of the of cell means. One procedure was nonparametric (Spearman Rank Correlation) and one was parametric (Pearson Correlation). These procedures compared experts to novices for each of the two dependent variables, and compared each group of subjects to itself for the two dependent variables.

4.2 Research Hypotheses

To find if experts differed from novices for the two time related dependent variables, the following null hypotheses were proposed. Null hypotheses Ho_1 and Ho_2 refer to the important blocks of the five graphs and null hypotheses Ho_3 and Ho_4 refer to the less important blocks of the graphs.

Ho_1: There is no significant difference between experts and novices in the percent of total time spent looking at blocks of a graph which contain important information.

Ho_2: There is no significant difference between experts and novices in their average fixation duration when looking at blocks of a graph which contain important information.

Ho_3: There is no significant difference between experts and novices in the percent of total time spent looking at blocks of a graph which contain less important information.

Ho_4: There is no significant difference between experts and novices in their average fixation duration when looking at blocks of a graph which contain less important information.

Ho_5: There is no difference between important and less important blocks in the correlation between and within novices and experts for their average fixation duration and percent of total time variables across the five graphs viewed.

Ho_6: There is no significant difference between novices and experts in their scores on the performance task.

4.3 Results

Table 1 contains the significance levels from the multivariate and univariate statistical tests for the important blocks of the five graphs used in the study. Table 2 contains similar information for the less important blocks of the five graphs.

TABLE 1

Significance Levels for Percent of Total Time and Average Fixation Duration by Block and Training for Important Blocks of the Viewed Graphs

Graph #	2		3		4		5		6	
	M	U	M	U	M	U	M	U	M	U
Training by Block	.119		.089		.401		.215		.139	
Percent Total Time		.825		.331		.063		.151		.046
Average Duration		.122		.256		.880		.886		.162
Block	.000**		.000**		.000**		.000**		.000**	
Percent Total Time		.000**		.000**		.000**		.000**		.000**
Average Duration		.000**		.000**		.000**		.000**		.000**
Training	.000**		.858		.044*		.000**		.016*	
Percent Total Time		.538		.994		.954		.005*		.300
Average Duration		.001**		.684		.038*		.000**		.005*

** $p < .005$ * $p < .05$ M = Multivariate   U = Univariate

TABLE 2

Significance Levels for Percent of Total Time and Average Fixation Duration by Block and Training for Less Important Blocks of the Viewed Graphs

Graph #	2		3		4		5		6	
	M	U	M	U	M	U	M	U	M	U
Training by Block	.192		.561		.403		.191		.277	
Percent Total Time		.175		.388		.832		.095		.061
Average Duration		.078		.249		.158		.202		.313
Block	.001**		.004**		.000**		.000**		.001**	
Percent Total Time		.001**		.094		.000**		.000**		.000**
Average Duration		.001**		.003**		.000**		.000**		.001**
Training	.122		.403		.157		.446		.231	
Percent Total Time		.850		.196		.374		.237		.220
Average Duration		.167		.637		.060		.256		.932

** p < .005   * p < .05   M = Multivariate   U = Univariate

Results of the analysis indicated that there were no significant training-by-block interactions for any of the five graphs. This was true for both the important blocks and the less important blocks of the graphs. Testing for block and training main effects proceeded in the absence of significant interaction between the main effects variables.

Multivariate tests for the block main effect showed significance for all graphs in both the important blocks and less important blocks categories. Univariate tests for percent of total time and average fixation duration within the main effect for block showed significance in all cases but one. For the less important blocks of graph #3, the univariate test of percent of total time failed to achieve significance.

Training was the variable of main interest. Results of the analysis for the training main effect showed differences between the two categories of blocks and the two dependent variables. In the analysis of the important blocks of the graphs (see Table 1), multivariate tests showed significance for four of the five graphs. Only graph #3 failed to achieve significance. Examination of the univariate results for these four graphs showed that in all four cases the average fixation duration variable achieved significance. The percent of total time variable achieved significance only for graph #5.

The results were different for the less important blocks of the graphs (see Table 2). In the analysis of these blocks, none of the multivariate tests for the training main effect achieved significance. Interpretation of the univariate tests was not appropriate in light of this lack of significance. However, examination of these univariate results showed that neither of the dependent variables achieved significance for any of the five graphs.

4.4 An Example

Figure 7 shows an example of one of the graphs (graph #5) showing the grid system of blocks and the blocks used in the analysis. The important blocks of this graph are blocks 47, 48, 49, 50, 51, 71, and 72 (indicated by an asterisk). These blocks contain the three axes intercepts, the origin (point (0,0)), scale values, and information about the minimum value of the graph. The less important blocks of this graph are blocks 28, 39, 40, 46, 52, 61, and 83. These blocks contain information about the scale of the axes in areas not as critical for interpretation of the graph. Appendix B, Figures 29 through 32, contains similar representations of the other four graphs used in the analysis.

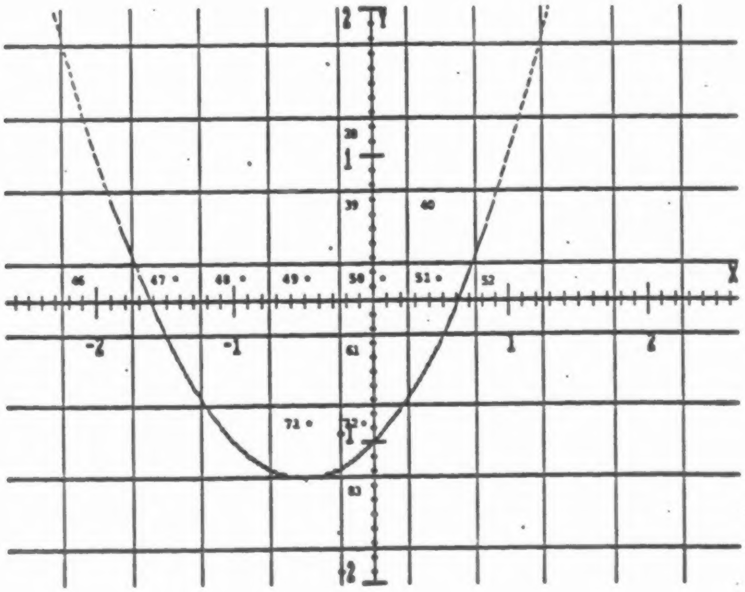


Figure 7: Graph #5 showing important and less important blocks.

Significance levels for the training main effect for graph #5, given in Table 1, indicate that significant differences were found between experts and novices for both percent of total time (p = .005) and average fixation duration (p = .000) when viewing the important blocks. Table 3 shows the value of those differences for each important block for both dependent variables. The differences between the groups of subjects (in the column marked "Diff.") are all positive, except for the percent of total time in block 47. The positive values indicate that the experts had higher mean values than the novices. The magnitude of the differences gives an indication of the size of the differences between the groups. These differences were significant for both variables.

Table 3

Cell Means and Differences
for Average Fixation Duration and Percent of Total Time
for Important Blocks of Graph #5

	Average Fixation Duration			Percent Total Time		
	Exp.	Nov.	Diff.	Exp.	Nov.	Diff.
Block 47	.402	.284	+.118	10.439	12.985	-2.546
Block 48	.356	.285	+.071	7.755	5.779	+1.976
Block 49	.301	.131	+.170	7.751	2.175	+5.576
Block 50	.490	.422	+.068	10.741	10.064	+0.677
Block 51	.479	.391	+.088	15.915	12.879	+3.036
Block 71	.241	.129	+.112	4.167	1.506	+2.661
Block 72	.436	.366	+.070	13.974	9.929	+4.045

Figures 8 and 9 are graphical plots of the cell means found in Table 3 for the important blocks of graph #5. Plotting the cell means in this manner shows graphically the parallelism from block to block between experts and novices for the two dependent variables. Significant parallelism was indicated by the lack of significant interaction of the main effect for block and training. These plots also show the variation in the cell means from block to block which is indicated by the significant results for the block main effect.

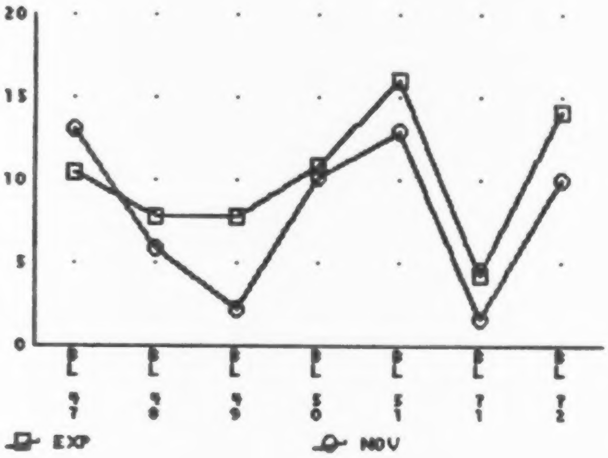


Figure 8: Plot of cell means for percent of total time of the important blocks of graph #5.

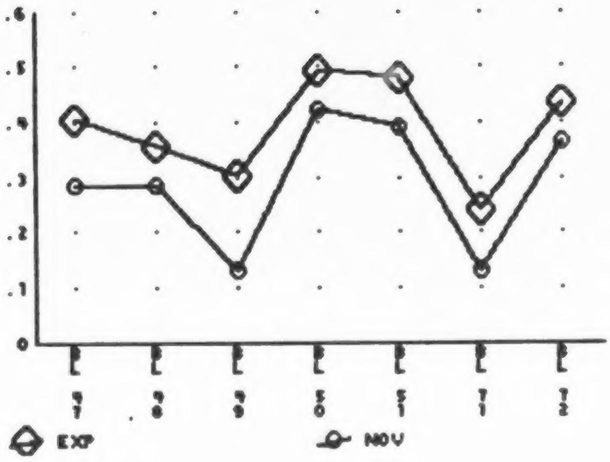


Figure 9: Plot of cell means for the average fixation duration of the important blocks of graph #5.

Significance levels for the training main effect for graph #5, given in Table 2, show no significant differences between experts and novices when viewing less important blocks of graph #5. This was true for both the multivariate and well as the univariate tests (multivariate: p = .446; univariate: p = .237 and p = .256). Table 4 shows the sign and magnitude of the differences between the cell means for these less important blocks. For each dependent variable, five of the seven difference values are negative, showing higher cell means for novices. The magnitude of the differences is relatively small since it is not significant.

Table 4

Cell Means and Differences
for Average Fixation Duration and Percent of Total Time
for Less Important Blocks of Graph #5

	Average Fixation Duration			Percent Total Time		
	Exp.	Nov.	Diff.	Exp.	Nov.	Diff.
Block 28	.084	.134	-.050	0.517	1.426	-0.909
Block 39	.085	.155	-.070	0.603	1.691	-1.088
Block 40	.042	.075	-.033	0.217	1.824	-1.607
Block 46	.140	.171	-.031	3.374	2.433	+0.941
Block 52	.200	.284	-.084	3.771	7.700	-3.929
Block 61	.218	.122	+.096	3.271	1.134	+2.137
Block 83	.131	.113	+.018	2.224	2.282	-0.058

Figures 10 and 11 show graphical plots for the cell means found in Table 4 for the less important blocks of graph #5. Again, the parallelism in the data is evident, as is the variation from block to block.

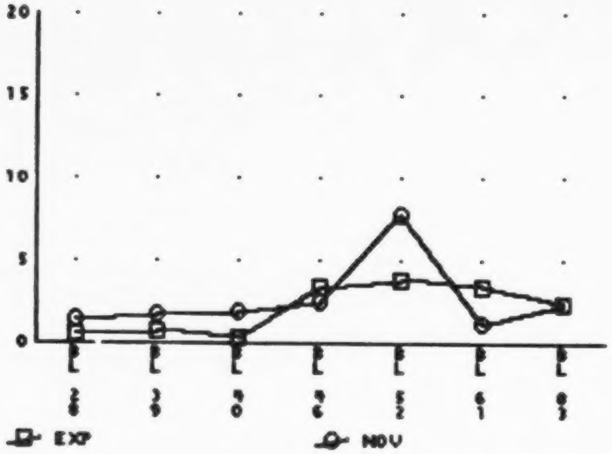


Figure 10: Plot of cell means for the percent of total time for the less important blocks of graph #5.

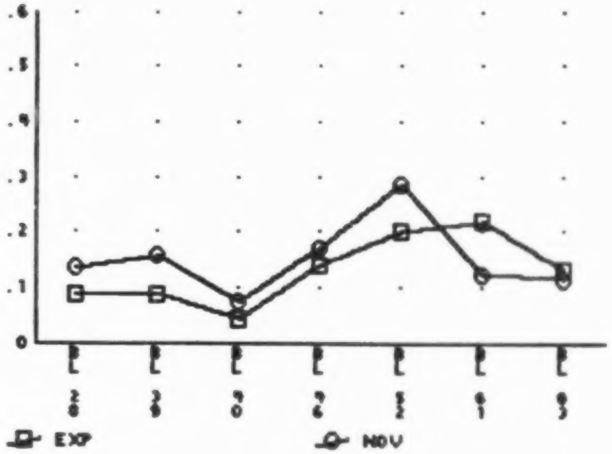


Figure 11: Plot of cell means for the average fixation duration of the less important blocks of graph #5.

Appendix C, Tables 11 through 18, contains numerical tables of cell means and differences for the two categories of blocks for graphs #2, #3, #4, and #6. As was the case with Tables 3 and 4 above, the difference between experts and novices for each of the two dependent variables is given as a signed number. These signed numbers indicate the direction and magnitude of the differences between the groups. Appendix B, Figures 33 through 48, contains graphical plots of the cell means from Tables 11 through 18.

4.5 Hypotheses Test Results

Null hypotheses Ho_1 and Ho_2 dealt with the important blocks category of the analysis. For graphs #2, #3, #4, and #6, null hypothesis Ho_1 was accepted. This indicated that when viewing important blocks, there was no significant difference between experts and novices for the percent of total time spent in blocks. Null hypotheses Ho_1 was rejected for graph #5, indicating a significant difference between groups of subjects in the percent of total time spent in the important blocks. Experts scored significantly higher for this variable (see Table 3).

Null hypothesis Ho_2 was rejected for graphs #2, #4, #5, and #6. The results indicated that significant differences were found between experts and novices for their average fixation duration in important blocks of the five graphs. In all cases, experts had significantly higher average fixation durations than novices (see Table 3 and Appendix C, Tables 11, 15, and 17). Null hypothesis Ho_2 was accepted for graph #3 only.

Null hypotheses Ho_3 and Ho_4 dealt with the less important blocks of the graphs. Both of these hypotheses were accepted for all five graphs. No significant differences were found between experts and novices for either the percent of total time or the average fixation duration when viewing less important blocks of the five graphs.

Table 5 contains a summary of the results of the hypotheses tests for null hypotheses Ho_1 to Ho_4 for the five graphs viewed. Hypotheses dealing with important blocks of the graphs are labeled with an asterisk (*). The other hypotheses dealt with less important blocks.

Table 5

Summary of Results of the Tests of Null Hypotheses Ho_1 to Ho_4 for the Five Viewed Graphs

Graph	2	3	4	5	6
Null Hypotheses					
* Ho_1	A	A	A	R	A
* Ho_2	R	A	R	R	R
Ho_3	A	A	A	A	A
Ho_4	A	A	A	A	A

A = Accept R = Reject

4.6 Results of the Performance Task

The performance task associated with this study was to select the correct graph from a set of five distractors and to write down important numerical information about the graph. Points were awarded based on selection of the correct distractor and the presence and correctness of numerical information about the graph. No points were awarded if the wrong distractor was chosen, even if the numerical information provided was correct. Possible scores were from 0 to 12 for each graph, or from 0 to 60 for each subject across the five graphs in the study. Actual scores ranged from 16 to 56 for the experts and 6 to 29 for the novices. Table 6 contains a summary of the statistics for the two groups of subjects.

Table 6

Means and Standard Deviations of the Performance Task for Experts and Novices

	Experts	Novices
Mean:	35.68	15.12
Standard Deviation:	13.68	5.75
Sample Size:	25	25

A T-test was performed comparing the two sample means. A value of t = 6.93 on 48 degrees of freedom was calculated.

Significance of this value was on the order of $p < .0005$. Experts scored significantly higher than novices on the performance task. Null Hypothesis Ho_{10} was rejected on the basis of this test.

4.7 Spearman Rank Correlation of Cell Means

The analysis of the data for important and less important blocks of the five graphs showed a strong pattern. The multivariate tests for the block main effect were significant in all ten analyses of important and less important blocks. Except for graph #3, all univariate tests showed significant differences between blocks for both the percent of total time and average fixation duration variables. As a measure of how these various blocks differ between experts and novices, a numerical ranking of the cell means of each block was done for both percent of total time and average fixation duration variables. These rankings were done for both important and less important blocks of each graph. The theory behind the ranking was the higher values for the two dependent variables indicated a higher degree of importance associated with that block. For example, on the important blocks of graph #2, experts had the longest average fixation duration when viewing block 50 (see Appendix C, Table 11). The seven blocks examined were ranked in order as 50, 28, 17, 54, 52, 19, and 26. Novices had the longest average fixation duration when viewing block 28. Their seven blocks were ranked 28, 50, 17, 54, 52, 26, and 19.

This ranking gave some indication of the order of importance subjects attached to the different blocks. More total time spent, or longer average fixation durations indicated more processing time spent at a location. More processing time meant better understanding or recall of the details viewed. A high positive correlation between experts and novices indicated that time factors were allocated to given areas of the graph in a similar manner regardless of the level of training. A low correlation indicated that level of training made a difference when allocating viewing time to different blocks. A negative correlation indicated that profound differences existed for time allocation between experts and novices.

The number of a given block had no relationship to its relative value, but only represented its position in an array. The Spearman Rank Correlation test was used to test the strength of the correlation between the experts and novices for the rankings on the two dependent variables. In the analysis, experts were compared to novices for each dependent variables. For example, the average fixation duration ranking for the important blocks of graph #2 , as given above, were correlated with each other. The result was a Spearmen Rank Correlation Coefficient of .929 (p = .012). The same expert/novice correlation for the less important blocks of graph #2 yields a Spearman Rank Correlation Coefficient of .393 (p = .168).

Table 7 contains a summary of the Spearman Rank Correlation Coefficients and significance levels for the various between subjects comparisons. Significance levels indicate the probability that the actual correlation is different than zero. Significance levels less than .05 indicate a high probability that the actual correlation is not equal to zero, based on the observed data. Significance levels greater than .05 indicate a lack of sufficient evidence to support the claim that the actual correlations are different than zero.

Table 7

Spearman Rank Correlation Coefficients for the Comparisons Between Experts and Novices for Percent of Total Time and Average Fixation Duration

Graph	2	3	4	5	6
Percent Total Time					
Important Blks.	1.000	.933	.650	.679	.929
Significance	.007*	.004*	.033*	.048*	.007*
Less Impt. Blks.	.607	.286	.905	.536	.405
Significance	.069	.242	.008*	.095	.112
Average Fixation Dur.					
Important Blks.	.929	.750	.633	.964	.786
Significance	.012*	.017*	.037*	.009*	.019*
Less Impt. Blks.	.393	.429	.667	.464	.714
Significance	.168	.147	.039*	.128	.016*

* $p < .05$

Correlations in this comparison were all positive. Correlations between experts and novices for important blocks were significant for all five graphs. Correlations for less important blocks were not significant except for three cases (graph #4 for both dependent variables and graph #6 for average fixation duration). These results indicate that there was a high degree of similarity between experts and novices when viewing important blocks and less similarity when viewing less important blocks of the graphs.

As a measure of subjects' consistency between the two dependent variables, within subject comparisons were made using the same rankings of cell means. Each group of subjects was correlated to itself across the two dependent variables. For example, experts' scores on the percent of total time variable were correlated to their own scores for average fixation duration. High, positive correlations indicated that subjects allocated total viewing time and fixation duration time in the same manner. A low or negative correlation indicated that subject had little or no similarity in their allocation of the two time factors from block to block. Table 8 contains the Spearman Rank Correlation Coefficients and significance levels for this comparison of subjects to themselves.

Table 8

Spearman Rank Correlation Coefficients for the Comparisons Within Subjects for the Percent of Total Time and Average Fixation Duration

Graph	2	3	4	5	6
Experts:					
Important Blks.	.750	.917	.866	.857	.881
Significance	.033*	.005*	.007*	.018*	.010*
Less Impt. Blks.	.964	.607	.905	.893	.857
Significance	.009*	.069	.008*	.014*	.005*
Novices:					
Important Blks.	.893	.867	.967	.607	.952
Significance	.014*	.007*	.003*	.069	.006*
Less Impt. Blks.	.964	.929	.810	.464	.881
Significance	.009*	.012*	.016*	.128	.004*

* $p < .05$

These comparisons show how consistent groups of subjects were between the two dependent measures. For example, when viewing graph #4, the correlation for experts between scores on the percent of total time variable and the average fixation duration variable for the important blocks was .866 ($p = .007$) and for the less important blocks was .905 ($p = .008$). All the correlations in this group were positive. Correlations were all significant except for three cases (graph #3 for less important blocks and graph #5 for both important and less important blocks). This indicated three types of consistencies within subjects. Subjects were consistent within themselves across the two dependent measures, across important and less important blocks, and from graph to graph.

4.8 Pearson Correlation of Cell Means

As another measure of the relationship between experts and novices, Pearson Correlation Coefficients were calculated for the same cell mean data used for the Spearman Rank Correlation. The same comparisons between and within subjects for the two dependent variables were made. Important blocks were contrasted to less important blocks.

Table 9 contains the Pearson Correlation Coefficients and significance levels for the comparison between experts and novices for the two dependent variables. As with the previous correlations, significance levels indicate the probability that the actual correlation is different than zero. Significance levels less than .05 indicate a high probability that the actual correlation is not equal to zero, based on the observed data. Significance levels greater than .05 indicate a lack of sufficient evidence to support the claim that the actual correlations are different than zero.

Table 9

Pearson Correlation Coefficients and Significance Levels for the Comparisons Between Experts and Novices for Percent of Total Time and Average Fixation Duration

Graph	2	3	4	5	6
Percent Total Time					
Important Blks.	.995	.936	.892	.840	.885
Significance	.000*	.000*	.001*	.009*	.002*
Less Impt. Blks.	.477	.528	.899	.540	.451
Significance	.139	.112	.001*	.106	.095
Average Fixation Dur.					
Important Blks.	.902	.816	.861	.969	.814
Significance	.003*	.004*	.001*	.000*	.007*
Less Impt. Blks.	.374	.457	.606	.557	.502
Significance	.204	.151	.056	.097	.070

* $p < .05$

Positive correlations indicate agreement between experts and novices on the way time factors were allocated block by block. Negative correlations indicate opposite strategies for time allocation between groups of subjects. High correlations indicate that time allocation strategies are the same for both groups across a type of block. Low correlations indicate little or no agreement in time allocation strategies.

All correlation coefficients were positive. The correlations between experts and novices were significant for both percent of total time and average fixation duration for the important blocks of all five graphs. Correlations were not significant for the less important blocks in all cases except for the percent of total time in less important blocks of graph #4.

This correlational analysis also gave an indication of the reliability of the selection process for important versus less important blocks of the graphs. Between subjects correlations showed how the blocks group together. Important blocks had significant correlation for both time variables. Less important blocks did not have significant correlations.

Table 10 contains the correlation coefficients and significance levels for the *within* subjects comparisons of the cell means across important and less important blocks of the five graphs. Significance levels have the same meaning for this table as for Table 9 above.

Table 10

Pearson Correlation Coefficients and Significance Levels for the Comparisons _Within_ Subjects for Percent of Total Time and Average Fixation Duration

Graph	2	3	4	5	6
Experts					
Important Blks.	.605	.861	.891	.874	.936
Significance	.075	.001*	.001*	.005*	.000*
Less Impt. Blks.	.878	.789	.957	.915	.910
Significance	.005*	.017*	.000*	.002*	.000*
Novices					
Important Blks.	.874	.886	.931	.839	.964
Significance	.005*	.001*	.000*	.009*	.000*
Less Impt. Blks.	.938	.822	.899	.888	.924
Significance	.001*	.012*	.001*	.004*	.000*

* $p < .05$

Positive correlations meant that subjects were consistent in their time allocation strategy for percent of total time and average fixation duration. When they spent more total time in a block, their average fixation durations were higher. The reverse would also be true (ie. less total time and less average fixation duration). Negative correlations meant that subjects reversed their time allocation strategy, spending more total time when they had shorter average fixation durations and visa-versa.

Low correlations meant that subjects were not consistent within themselves, changing their viewing strategy as they looked at different important blocks or different less important blocks. High correlations meant that subjects had consistent viewing strategies for the total time and average fixation duration for the different blocks of each type.

All of the correlation coefficients were positive. Except for experts when viewing important blocks of graph #2, all correlation coefficients were significant. These results indicated that subjects had consistent viewing patterns within themselves for the two viewing time factors. This consistency was apparent for both important and less important blocks of all five graphs.

CHAPTER V

DISCUSSION AND RECOMMENDATIONS

5.1 Introduction

The results of this study indicated that there were both differences and similarities between experts and novices when reading mathematical graphs. Differences were found for the average fixation duration on important blocks and in the correlations between subjects for the less important blocks of the graphs. Similarities were found for the percent of total time spent viewing both important and less important blocks and in the correlations between subjects for the important blocks. Within-subjects correlations displayed similar time allocation strategies for both groups. Parallelism in the data showed similarities between groups of subjects from block to block and argued for the power of the symbol system to draw the viewer to important areas of the graph. Block by block variation in viewing time factors and the correlations between subjects gave evidence of local control of eye movement patterns for both experts and novices. Recommendations for changes in curriculum and instruction resulted from both the similarities and the differences between experts and novices.

5.2 Viewing Time Factors

For four of the five graphs viewed, there were significant differences between experts and novices for one of the two time factors when viewing important blocks of the graphs (see Table 1). There were no differences between experts and novices for either time factor when viewing the less important blocks of the five graphs (see Table 2). Experts did something different when they moved their eyes to the important blocks of the graphs. They had longer average fixations on the blocks which contained important information. Novices did not alter their fixation durations in the same manner when they viewed important information.

Even though the experts had significantly longer fixations in the important blocks of the graphs, this did not mean that they spent longer periods of time looking at the important information. Except for one graph (graph #5), there were no significant differences between experts and novices for the percent of total time that was spent on the important or less important blocks of the graphs (see Tables 1 and 2). The simple speculation that experts read graphs more effectively because they spend more time looking at the important information was not supported by the data. On the contrary, the evidence indicated that both groups allocated total viewing time equally for important blocks of the graphs. This was also true for the less important blocks.

Figure 12 contains a representation of the percent of total time and average fixation duration for a typical important block of a graph. The two circles represent the percent of total time spent in the block by experts and novices. The circles are of equal area. The lengths of the line segments in the circles represent the lengths of the average fixation durations of the two groups. Since total time in a block is the sum of the fixation durations, the sum of the lengths of the line segments in the "expert" circle is equal to the sum of the line segments in the "novice" circle. Lengths of the line segments differ, corresponding to the differences found between experts and novices for average fixation duration.

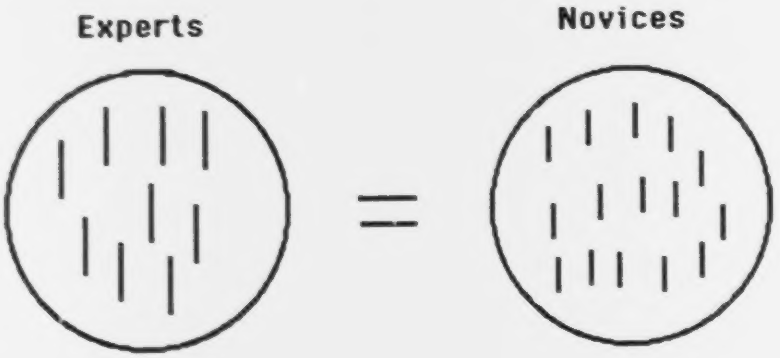


Figure 12: Viewing time for experts and novices in a typical important block.

Viewing time for the experts was made up of a smaller number of longer fixations. Viewing time for the novices was made up of a larger number of short fixations. These results related to the scores on the performance task. Experts were able to decode the graphs better as measured by the scores on the performance task. Experts scored significantly higher on the performance task than novices (p < .0005). Some of the reasons for the difference in these scores were due to factors other than average fixation duration. However, Loftus' (1981) study of average fixation duration and performance on memory tasks supported an interpretation directly linking performance on a task to fixation duration. As average fixation duration increased, performance on the memory task increased. The task in this study was a memory task. Experts had longer average fixation durations in important blocks and scored better on the performance task.

The analysis of the less important blocks showed a different result. Figure 13 contains a representation of the percent of total time and average fixation duration for a typical less important block of a graph. In this case, there were no differences between the two groups for either of the two time factors.

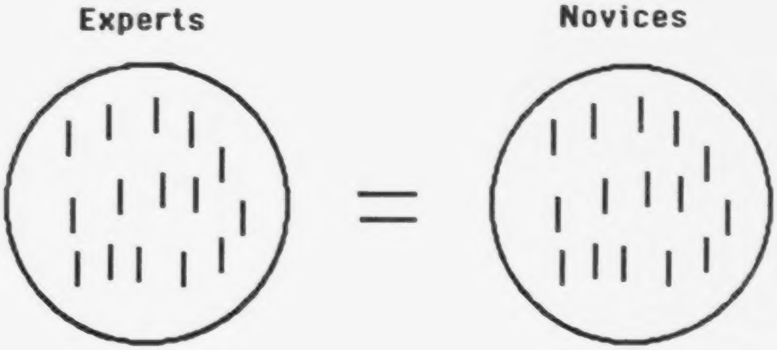


Figure 13: Viewing time for experts and novices in a typical less important block.

When processing less important areas of the graphs, there was no difference between experts and novices for percent of total time or average fixation duration. Experts differentiated between important and less important blocks of the graph and adjusted their fixation times accordingly. The simple suggestion that experts do not waste their total viewing time in less important areas of the graph is not supported by the data. There was no difference between the two groups in total time allocation for important or less important blocks of the graphs. It was the average fixation duration in important blocks which defined an important difference between experts and novices.

5.3 Correlation Results

Where the multivariate analysis of the data tested for differences in magnitude between experts and novices, the correlational analysis tested for proportional relationships between the groups of subjects. Given the relative magnitudes of the cell means, what was the nature of the relationships between experts and novices for the two dependent variables? The results of the correlational analysis (both parametric and nonparametric) showed some strong patterns.

Correlation of the ranks of cell means (Spearman Rank Correlation) and the cell mean values themselves (Pearson Correlation) showed that experts and novices had highly similar viewing time allocation strategies when attending to the important blocks of the graphs. This was true for both percent of total time and average fixation duration across all five graphs (see Tables 7 and 9). In important blocks where experts' average fixation durations were relatively high, novices' average fixation durations were also relatively high. In important blocks where experts had a relatively low average fixation duration, novices were also relatively low. The same relationship occurred for the percent of total time. While there was this parallelism between groups, the experts were significantly higher than the novices for only the average fixation duration variable.

When attending to less important blocks, experts' viewing patterns were not the same as those of novices. Except for the percent of total time on graph #4, the parametric correlation (Pearson Correlation, Table 9) showed no significant correlations between expert and novice time allocation strategies. Nonparametric correlations (Spearman Rank Correlation, Table 7) showed the same lack of significance except for graph #4 (correlation of both dependent variables were significant) and graph #6 (correlation of average fixation duration was significant). The multivariate analysis of the less important blocks showed no significant differences in the magnitudes of the percent of total time or average fixation duration, but the correlational analysis showed that there were differences in viewing time allocation strategies. For example, when viewing some less important blocks, novices allocated more percent of total time than experts even though the magnitudes of the percent of time was not significantly different. The reverse was also true.

5.4 The Power of the Symbol System

Why were experts and novices similar in time allocation strategy for the important blocks and dissimilar for less important blocks? The answer lies in the power of the symbol system to draw the graph readers' eyes to the important areas of the graph. Level of experience affected only the absolute magnitude of the average fixation duration in important blocks, while the relative magnitudes and rankings of the cell means for the important blocks were the same for experts and novices. Novices were attending to the important blocks of the graphs and allocating viewing time in those blocks in the same pattern as experts, even without a base of experience. The graphs had the power to draw novices to important areas in the same relative pattern as experts. But, once novices got to the important areas, they did not change their viewing strategy by increasing average fixation durations as experts did.

The power of the graph to draw the eye to important areas was different from the cognitive decision to increase fixation duration. The cognitive decision was related to a subject's experience and background. If the important areas of the graphs were accounting for both the spatial decisions and the temporal decisions in eye movement patterns, then there would have been no difference between subjects for the average fixation duration in important blocks. Novices were drawn to the important areas of the graphs, but did not make the cognitive decision to increase fixation duration for more effective processing. The graphs helped novices make the decisions of where to look, but not the cognitive decision to concentrate longer on the important blocks as the experts did.

Less important blocks of the graph did not draw experts' and novices' eyes in the same way. There was no significant difference between the magnitudes for percent of total time or average fixation duration, and patterns of time allocation (both rank and cell mean) were not significantly correlated. This indicated that less important blocks did not draw novices' eyes in an order like experts.

Pinker's (1981) theory suggested that the power of a graph to transmit information depended on the suitability of the type of graph to the question being asked and the experience of the graph reader. Significant correlations between experts and novices for the important blocks of these cartesian graphs supported Pinker's theory for one type of graph. Cartesian graphs were effective in displaying the mathematical relationship between two variables because they can draw the reader to the important information. Once the reader moved his/her eyes to the important information, the experience of the reader guided the cognitive decision of fixation duration.

Cleveland and McGill's (1984) theory attempted to define a hierarchy of perceptual tasks which made graphs more or less understandable. The power of a cartesian graph came from the fact that points of the graph were interpreted by their position along common scales in both the horizontal and vertical direction, and their relationship to each other through the point to point and global rates of change (slope). Support for this theory was found in the way readers were drawn to the important blocks of the graphs. Most of the important blocks of the graphs from this study contained information about the axes, which are the common scales. Both experts and novices were drawn to these important blocks in the same pattern. The graphs were understandable by reference to the axes.

5.5 Local Control of Eye Movement Patterns

The power of the graph to draw the eye to important areas of the graph does not preclude the issue of local control of fixation duration. Global control of eye movement patterns would indicate little or no difference in fixation duration from one area of the visual field to another. Local control would indicate eye movement patterns which responded to the type of information present in a given block. Experts were able to respond to the information in different areas of the visual field and increase or decrease their average fixation durations. Novices did not show the magnitude of change in average fixation durations, but the parallelism in the data and the correlations between groups showed that they responded to changing information in the different important blocks in the same pattern as experts. The cognitive decision to increase or decrease fixation durations was separate from the spatial decision which was influenced by the graph being viewed.

This study did not investigate the spatial decisions related to fixation sequence (scanpaths). This spatial decision was different than the decision of where to look. Fixation sequence was a cognitive spatial decision more closely related to the temporal decision of fixation duration. Investigating the relationship between graph interpretation and fixation sequence will provide more information about local control of eye movement patterns. This will be the topic of future research.

5.6 Consistency Within Subjects

Within-subjects correlations showed how subjects compared within themselves for the two time factors. For example, when experts viewed graph #6, the correlation between percent of total time and average fixation duration for the important blocks was .936 ($p < .001$; see Table 10). This meant that when experts had a high average fixation duration, they also had a high percent of total time in the important blocks of graph #6. The reverse was also true (ie. low average fixation duration meant low percent of total time). Experts were not spending relatively large percents of their viewing time in blocks where they had relatively short average fixation durations, nor did they have relatively short fixation durations in blocks where they spent relatively large percents of total time.

This significance pattern of within-subject correlations for the important blocks was present across all graphs for both the parametric and nonparametric correlations. Only novices viewing graph #4 failed to reach significance for the Spearman Rank Correlation ($p = .069$; see Table 16). For the Pearson Correlation, only experts viewing graph #2 failed to reach significance ($p = .075$; see Table 18). Both of these exceptions were marginal.

Correlations within subjects for less important blocks were also significant. Except for the Spearman Rank Correlation of novices viewing graph #5, all correlations were significant (see Tables 8 and 10). This meant that even for less important blocks of the graphs, both novices and experts had a direct relationship between the relative percent of total time and average fixation duration in different blocks.

Subjects were consistent in the time allocation strategies within themselves across all five graphs for both important and less important blocks. This pattern of consistency had two interpretations. First, there was a predetermined cognitive time allocation pattern which subjects used to interpret graphs of the type in the study. This was similar to the scanpath theory of Norton and Stark (1971) and indicated a lack of local control over eye movement time patterns. Second, local control of the viewing time factors affected both percent of total time and average fixation duration in the same way and in relatively the same amounts. For example, when novices viewed a block, if they had a relatively high percent of total time in that block, then they had a relatively long average fixation duration. The information in the block affected both time factors in the same way.

If predetermined cognitive patterns for viewing time were being used, then those patterns would be global, showing little variation from block to block. Global theories of text processing stated that average fixation duration remained relatively constant across an entire passage. Evidence showed correlations between experts and novices for the important blocks were significant. The multivariate analysis showed that there were significant differences between important blocks for both percent of total time and average fixation duration (see Table 1, "Block" main effect). These two pieces of evidence taken together indicated that a global pattern for time factors was not being used by subjects for the important blocks. If there were no global patterns for the important blocks, then there were no global patterns for less important blocks since a pattern for one type of block and not for the other would indicate local control.

Evidence indicated that local factors in different areas of the graphs directly affected average fixation duration for both experts and novices. Local control for experts was apparent because of the increased average fixation duration on important blocks. Local control for novices was indicated because of the strong correlation between experts and novices for important blocks.

The within-subjects correlation showed similarity between graphs. All of the graphs affected the viewing time factors within subjects in the same way. None of the graphs, for example, caused the subjects to display an inverse relationship between the two time factors.

5.7 Recommendations

The main purpose of this study was to identify variables critical to graph interpretation as a first step in designing better curriculum materials and methods of instruction. The results of the study showed that the experience of the graph reader played a major roll in the interpretation process by signaling the need for the cognitive decision to increase fixation duration in important blocks of the graph. The mathematical graphs used in the study had the ability to draw experts' as well as novices' eyes to the important information. These two factors indicated two specific recommendations:

1. Students should be given substantially more experience in reading, interpreting, and using information in graphical form. Experiences using mathematical graphs should begin at earlier levels and be more intense, involving both curriculum and teaching methods.

2. Mathematical graphs are understandable and appropriate for even novice users, and should be used in more instances throughout the curriculum.

The ability to implement these recommendations in the schools at the present time is within reach. The best way for students to get more experience in using graphs is to provide them with the means to create and manipulate many graphs with relatively little effort. Many teachers shy away from using mathematical graphs for concept development or problem solving because of difficulty in creating and manipulating many instances of graphs. Point plotting can be a time consuming task. For a student to create a useable, accurate graph often implies the student already understands the concept or problem being studied. Relying on printed material to display the exact mathematical graph needed in a given teaching situation is often disappointing.

Mathematical graphs need to become a teaching and learning tool which can be used in any situation, can be tailored to the specific problem at hand, and can be manipulated easily and efficiently to illustrate different situations. Computers and graphing calculators (Casio, Sharp, and Hewlett-Packard) can create accurate, useable, and easily manipulated mathematical graphs. These machines can do for graphs what hand-held calculators have done for arithmetic. They make concepts and skills accessible earlier and give students a base of experience through multiple exposure to many different types of graphs. When solving problems, Polya's (1945) general problem solving technique of drawing a picture to help understand the problem can now be extended to include drawing a graph to understand the relationship between the variables in the problem. Students and teachers now have the ability to exploit the human visual processing system to add another dimension to the teaching and learning of mathematics.

In most school systems, personal computers are available. Good graphing software is available, or programs can be written for specific instances. Professional journals like The Mathematics Teacher often have articles for teachers using graphs for concept development or problem solving; computer programs are usually provided. Graphing calculators are relatively inexpensive and easy to use. As research and development costs are recouped and competition in the market increases, costs for these calculators will be further reduced.

The hardware and software are in place or readily available. The missing element is the integration of the new methods into curricular materials to be used in the schools. This step is not as easy as it seems. Curricular change is slow to say the least. Innovative teachers will recognize the benefits of this graphing technology and make their own changes. Other teachers will rely on textbooks for guidance in what and how to teach. Often the only effective means of changing teaching methods is through attrition. Pre-service teachers need to be taught the importance and effectiveness of using graphs as teaching and learning tools.

Textbook publishers need to be encouraged to include more graphical learning methods in their books, and to integrate the new technology in effective ways.

The mathematical graphs used in this study had the power to draw novices' eyes to the important information. While these novices were not 11 or 12 year olds, their mathematical knowledge and experience was limited. They responded to important information in the graphs in a manner similar to experts. Mathematical graphs could be used more extensively in the pre-algebra mathematics curriculum at the middle school/junior high level for concept development and problem solving. Research found that students were good at plotting points, a common early graphing experience. Early graphing experiences need to be extended to include graph reading and interpretation skills at this level.

Common Algebra I and II courses provide many opportunities for interactive use of mathematical graphs. For example, when learning about the slope of the graph of a linear equation, students could experiment with many examples to develop an understanding of linear graphs with slopes greater than one as compared to linear graphs with slopes between zero and one. Or, when studying functional transformations such as "$f(x) = x^2$" transformed to "$f(x) = (x - 2)^2$," students could experiment to help develop the concept themselves.

Higher level mathematics in high school and college have just begun to exploit the modern technology and interactive graphing for teaching and learning. New techniques for solving problems, understanding relationships, and developing concepts are being used. For example, solving a cubic equation for its roots can be done with a graphing calculator by "zooming in" on a root and estimating its value. This estimation can be as accurate as the machine will allow, usually 8 to 10 decimal places.

Mathematics instruction at all levels can benefit from increased use of mathematical graphs. Experience with mathematical graphs should begin earlier and be more intense in the curriculum. This experience should be expanded to include interpretation of graphs and not just point plotting. Like reading, novice graph readers become more adept in reading and understanding mathematical graphs by having more experience with them.

5.8 Future Research

Future studies of graph interpretation should build on the findings of this study for polynomial graphs and should branch out to include methods of instruction, problem solving techniques, other types of graphs and other populations. Investigations should include analyses of fixation sequence (scanpaths) for mathematical graphs. Data graphs of several types (bar, pie, cartesian, etc.) should be investigated to see if the graphical interpretation theories are supported by eye movement data.

Techniques of instruction using mathematical graphs in concept development should be compared to non-graphical methods. Problem solving using graphs could be studied using an expert/novice comparison similar to this study. Methods of teaching problem solving skills with and without graphs could be contrasted. Other age groups should be studied to see if the results of this study generalize to younger students, or if there are other similarities and differences yet to be discovered.

Each of these studies has implications for curriculum and instruction in mathematics classrooms. Some studies have more direct relationships to the classroom, but each study will advance our knowledge of how to teach and use mathematical graphs in the classroom and in the real world.

APPENDIX A

FIGURES 14 THROUGH 28 REFERRED TO IN CHAPTER III

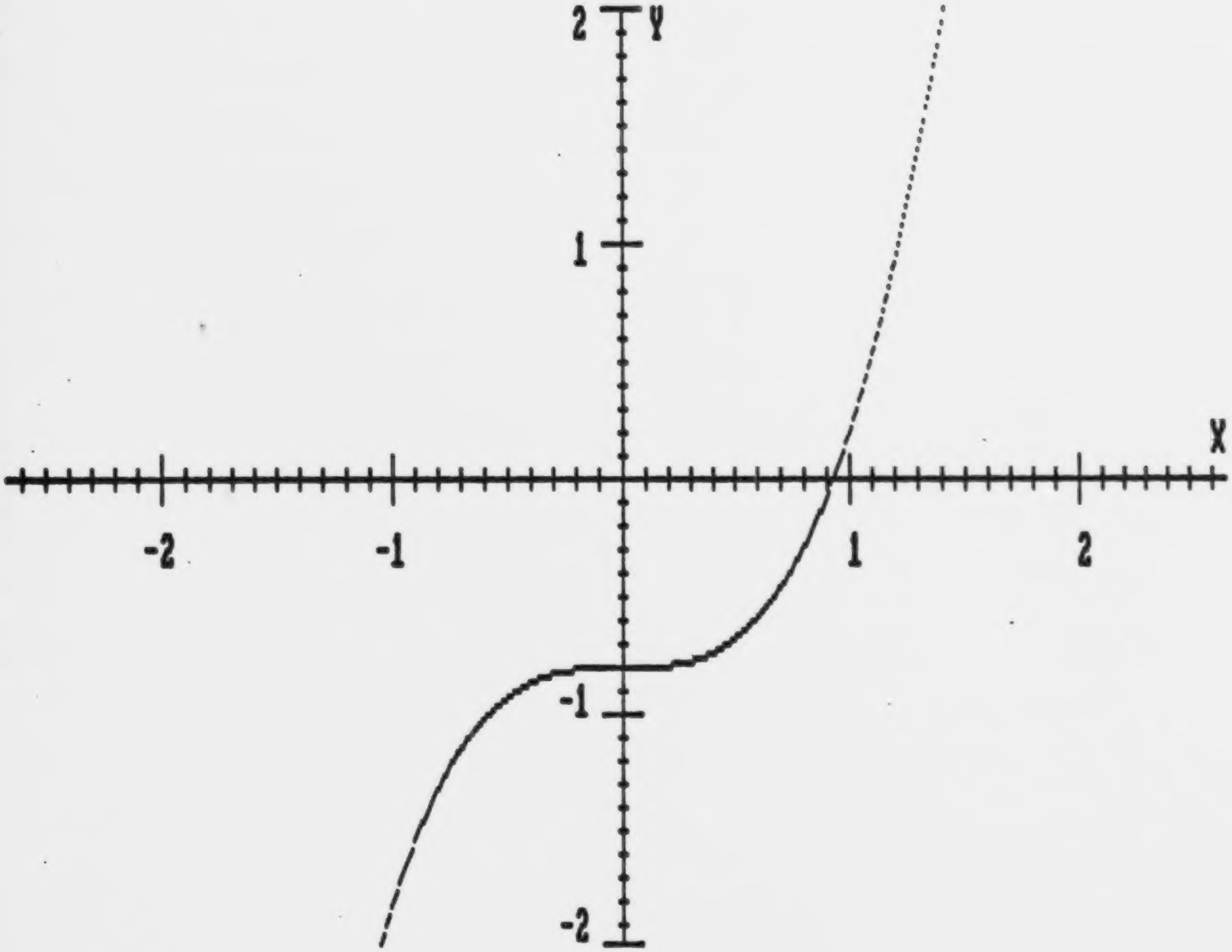


Figure 14: Screen image of graph #1.

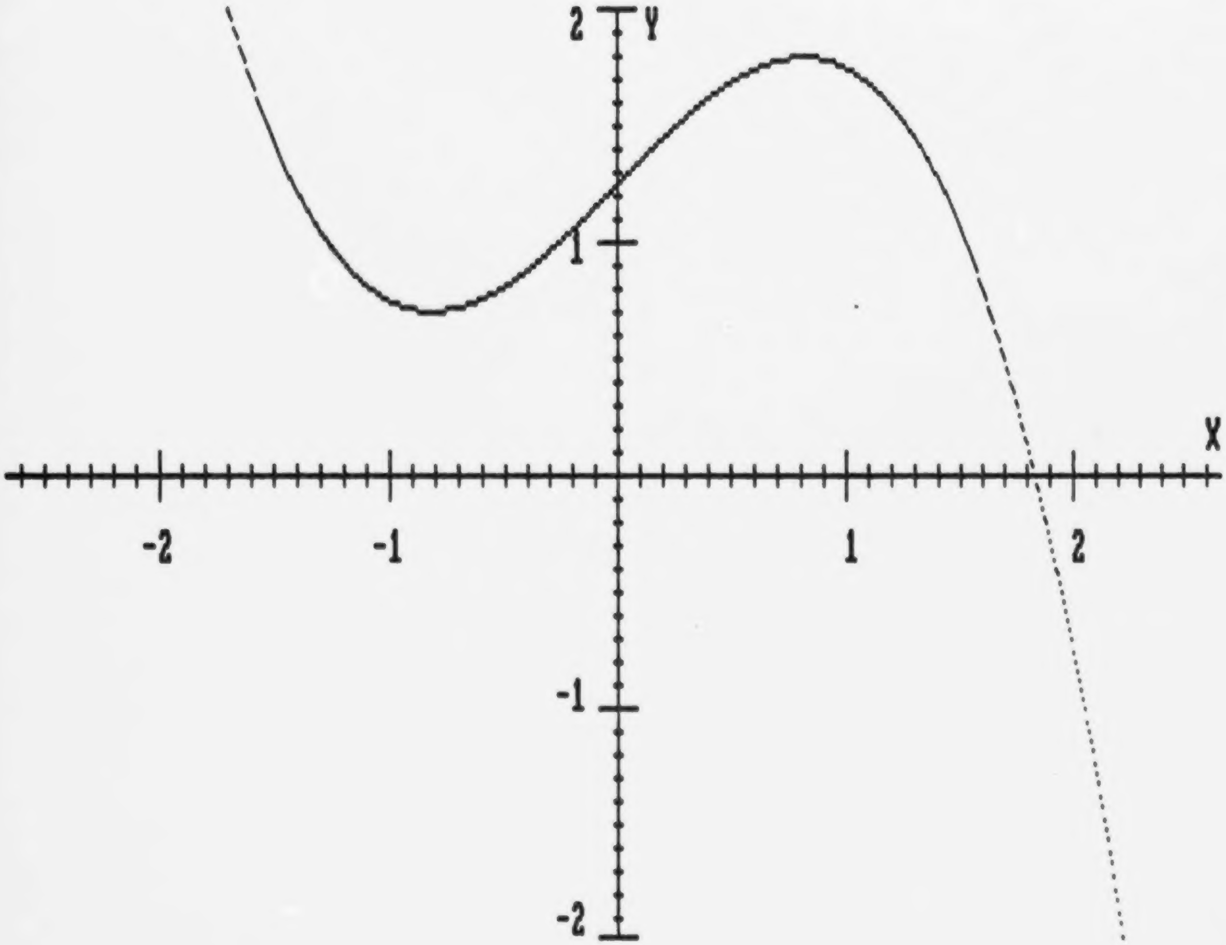


Figure 15: Screen Image of graph #2.

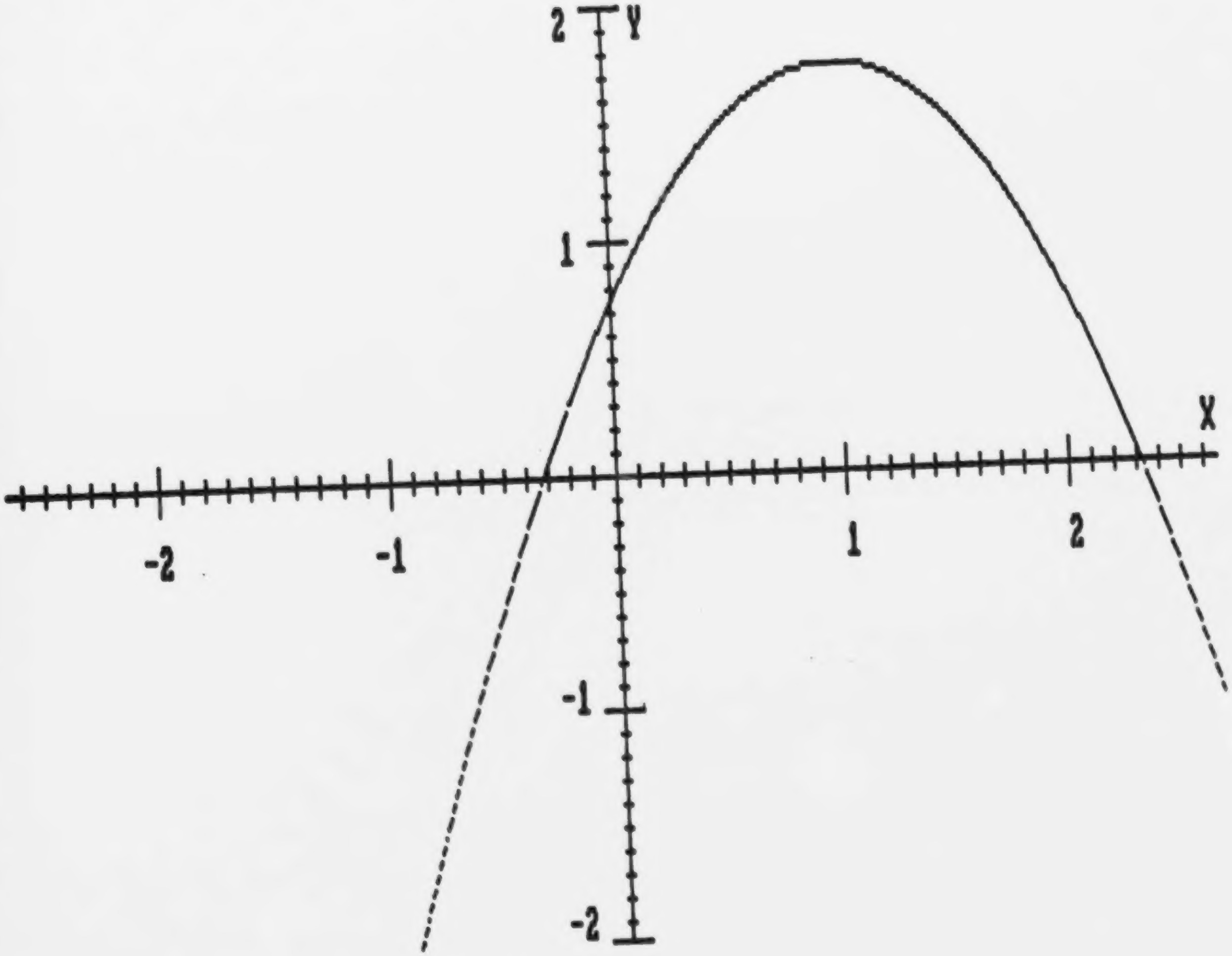


Figure 16: Screen image of graph #3

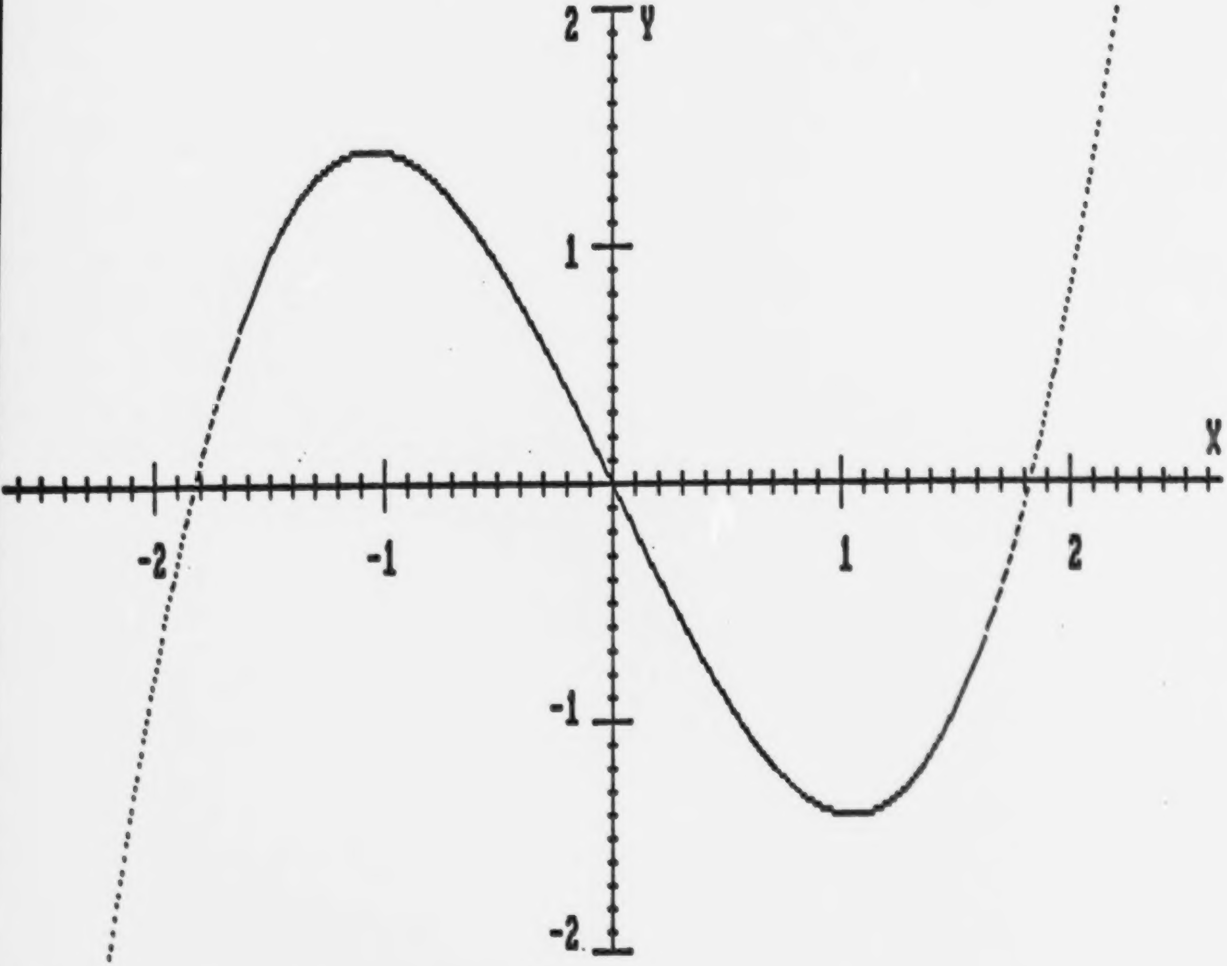


Figure 17: Screen image of graph #4.

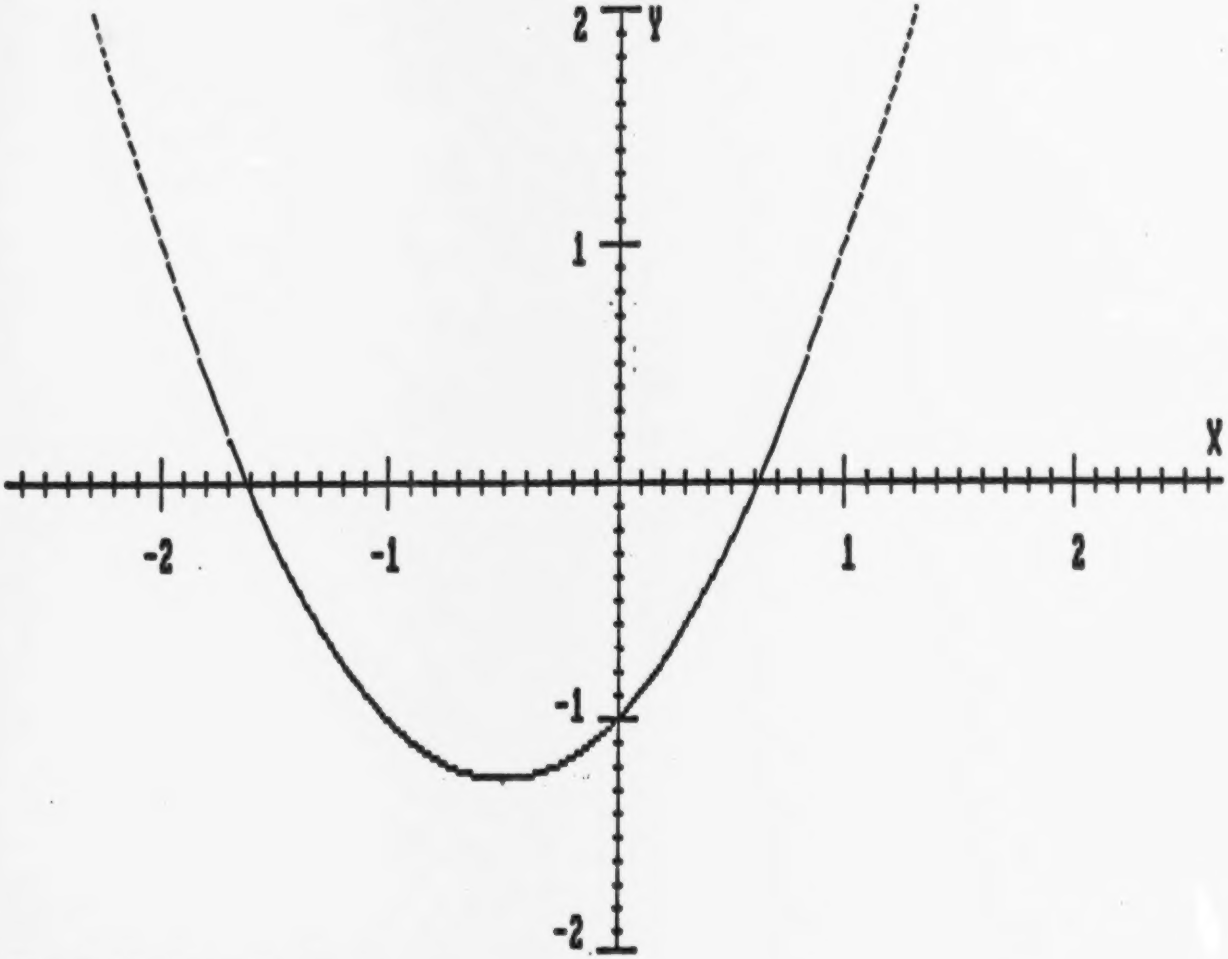


Figure 18: Screen image of graph #5.

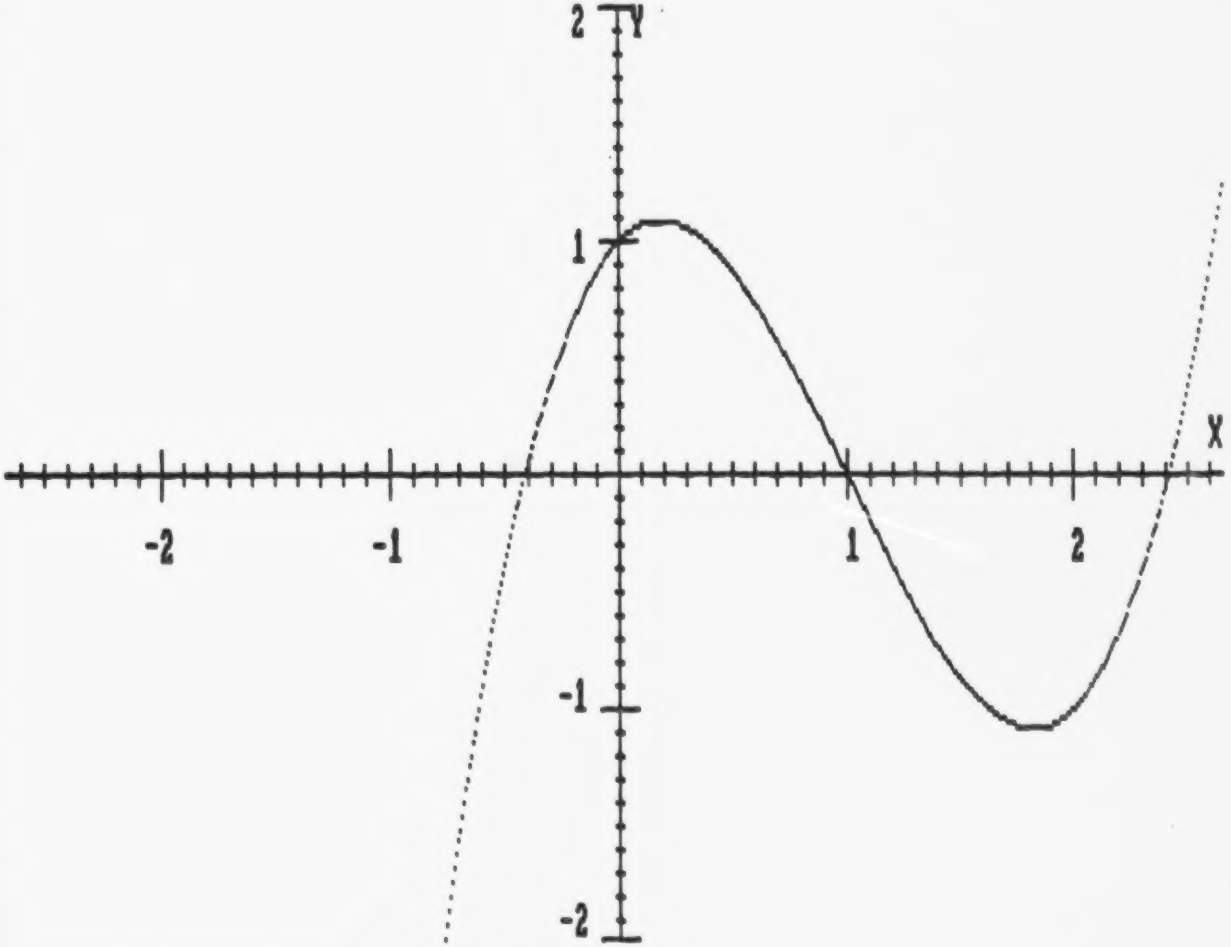
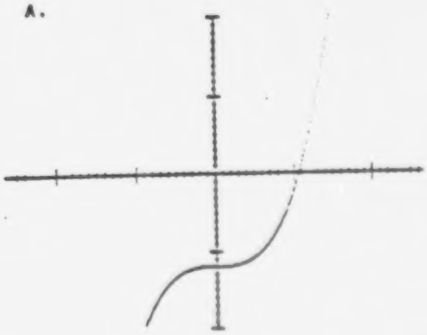
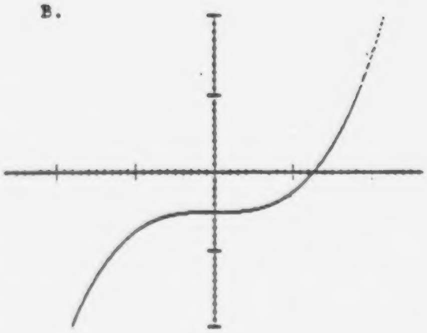


Figure 19: Screen image of graph. #6.

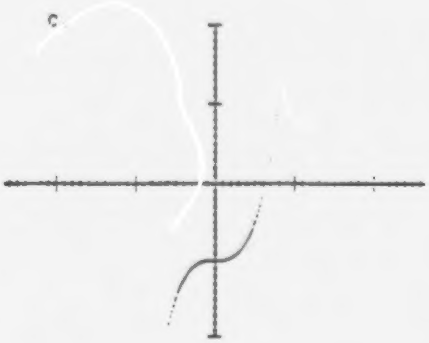
A.



B.

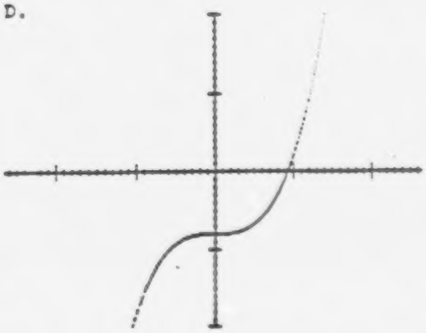


C.



Subject: ________

D.



E.

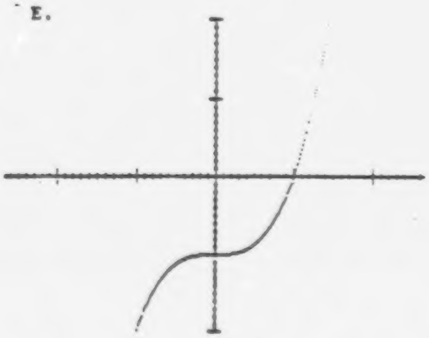
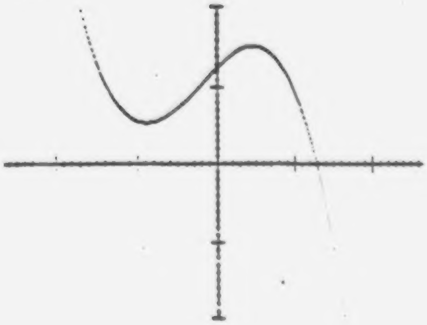


Figure 20: Multiple choice distractors for graph #1.

A.



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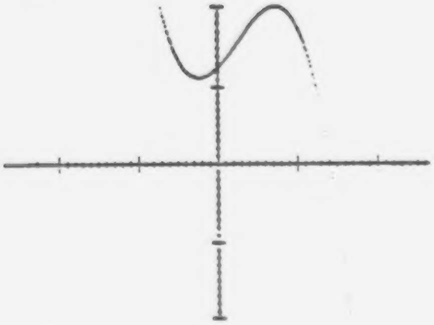

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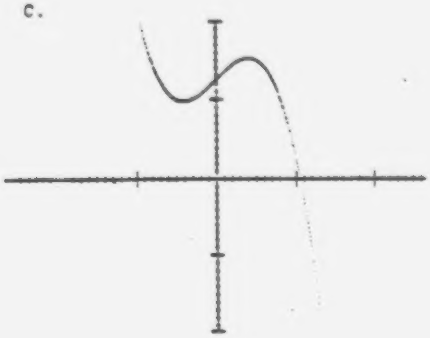
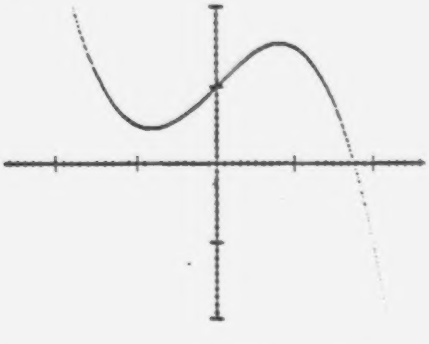


Subject: ______

D.



E.

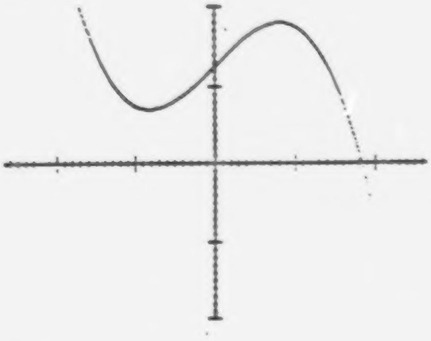
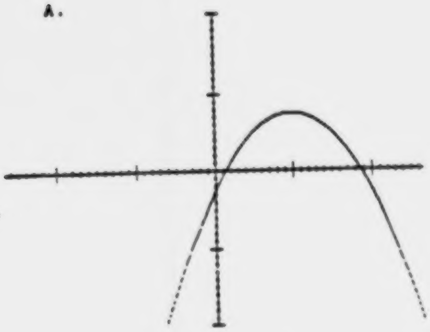
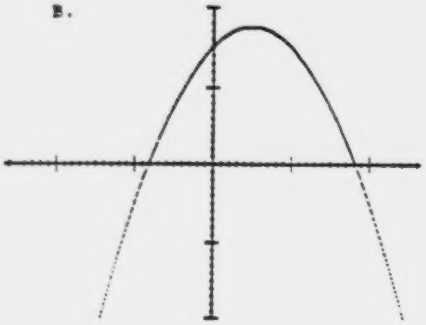
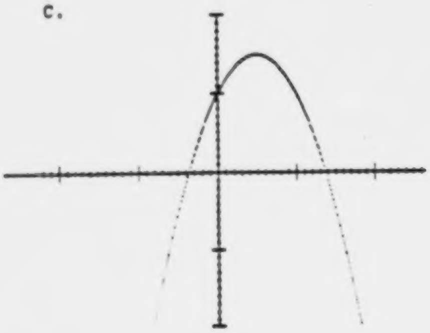


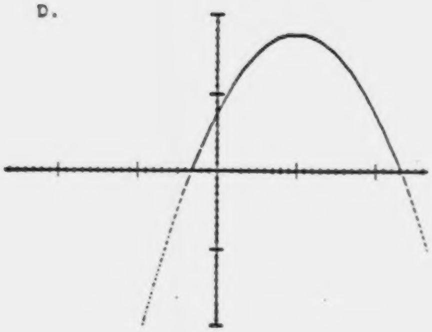
Figure 21: Multiple choice distractors for graph #2.







Subject: ______



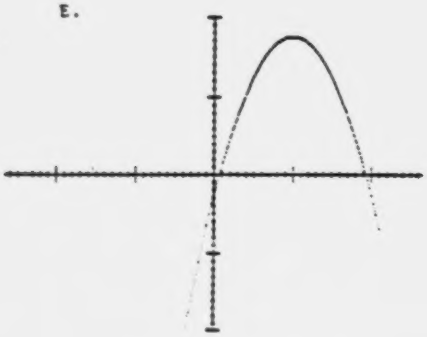
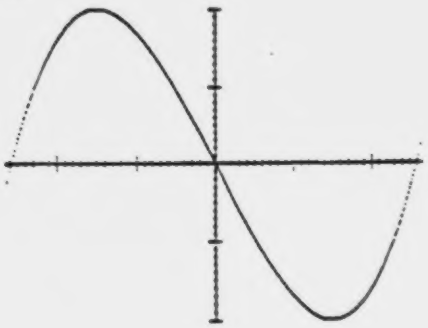
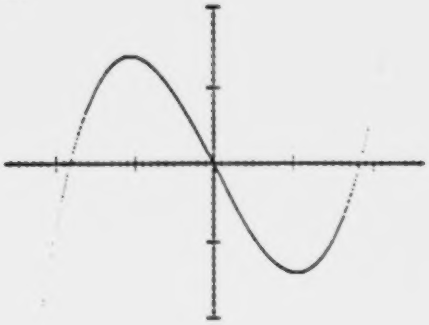


Figure 22: Multiple choice distractors for graph #3.

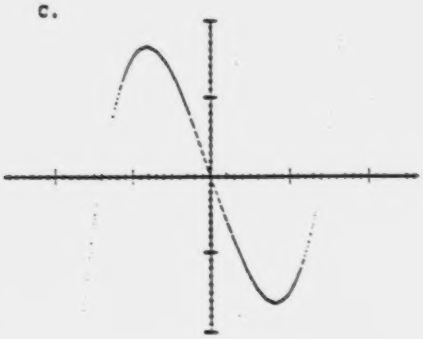
A.



B.

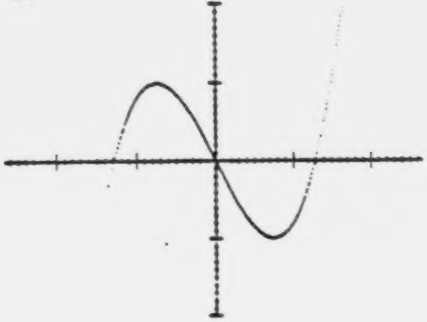


C.



Subject: ______

D.



E.

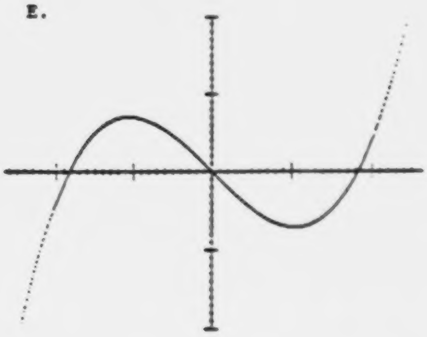
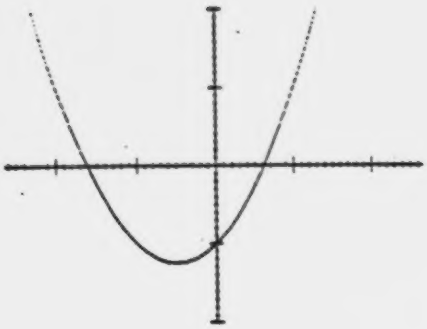
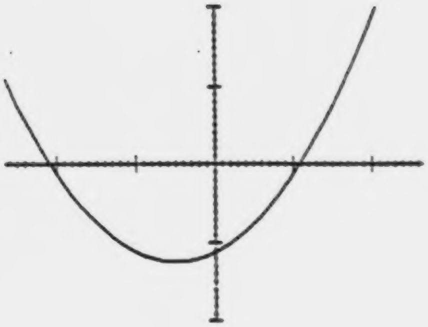


Figure 23: Multiple choice distractors for graph #4.

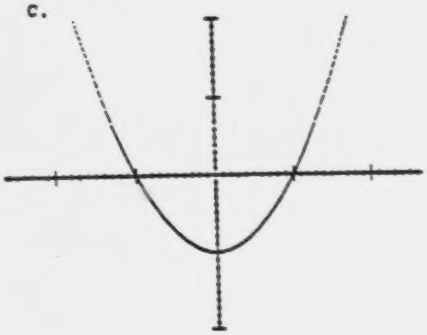
A.



B.

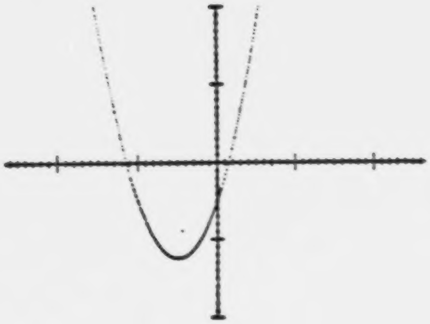


C.



Subject: ________

D.



E.

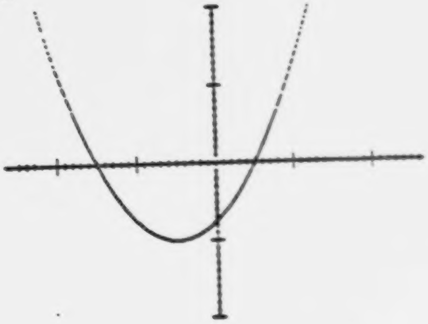
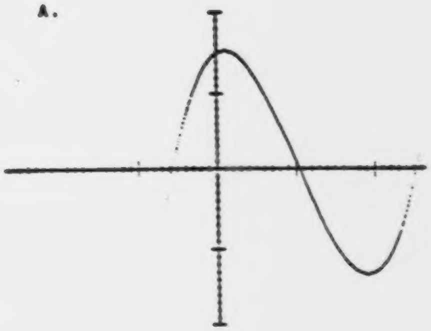
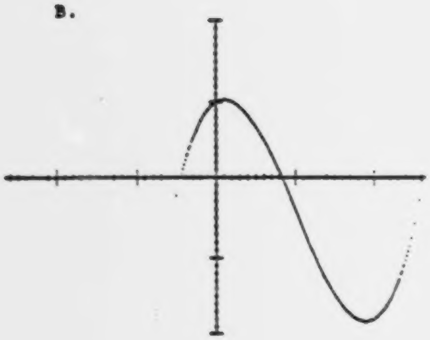


Figure 24: Multiple choice distractors for graph #5.

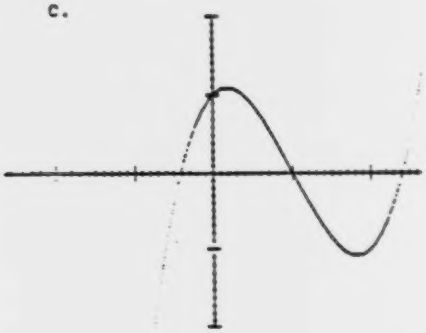
A.



B.

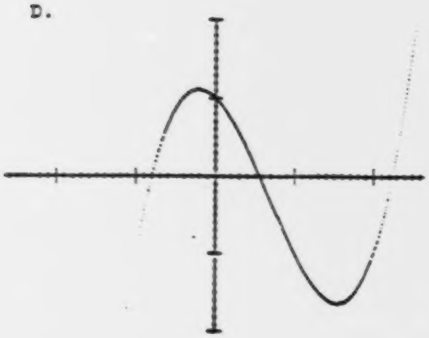


C.



Subject: ______

D.



E.

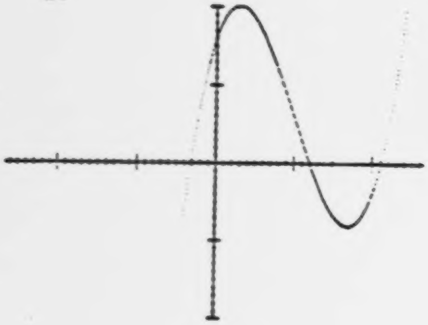


Figure 25: Multiple choice distractors for graph #6.

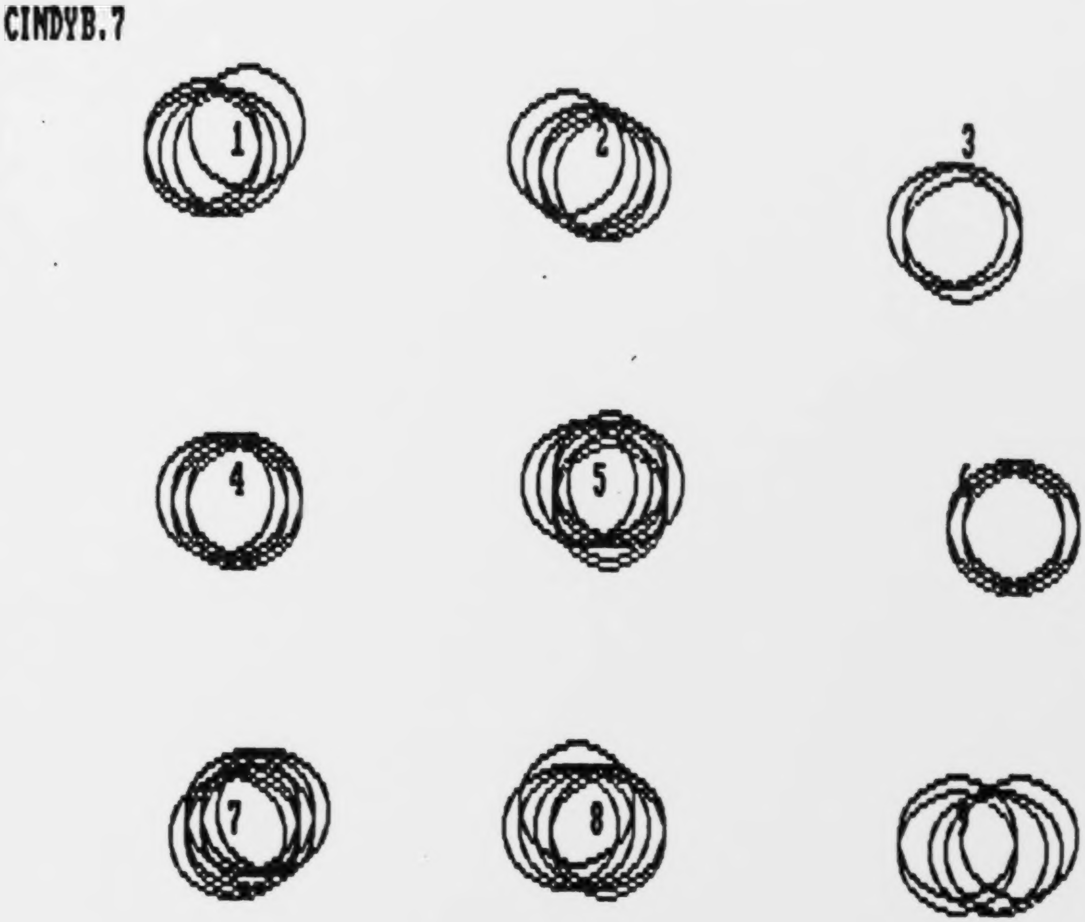
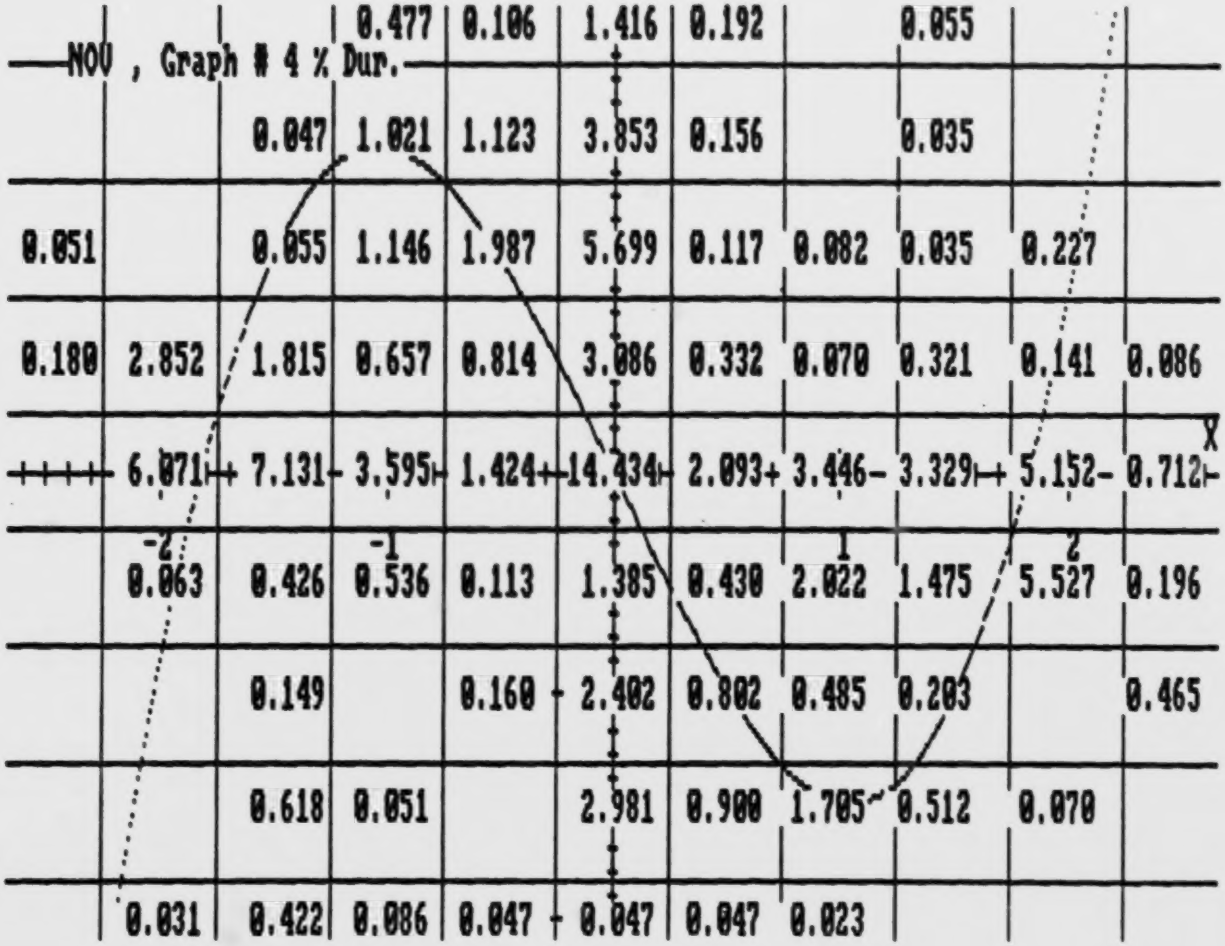
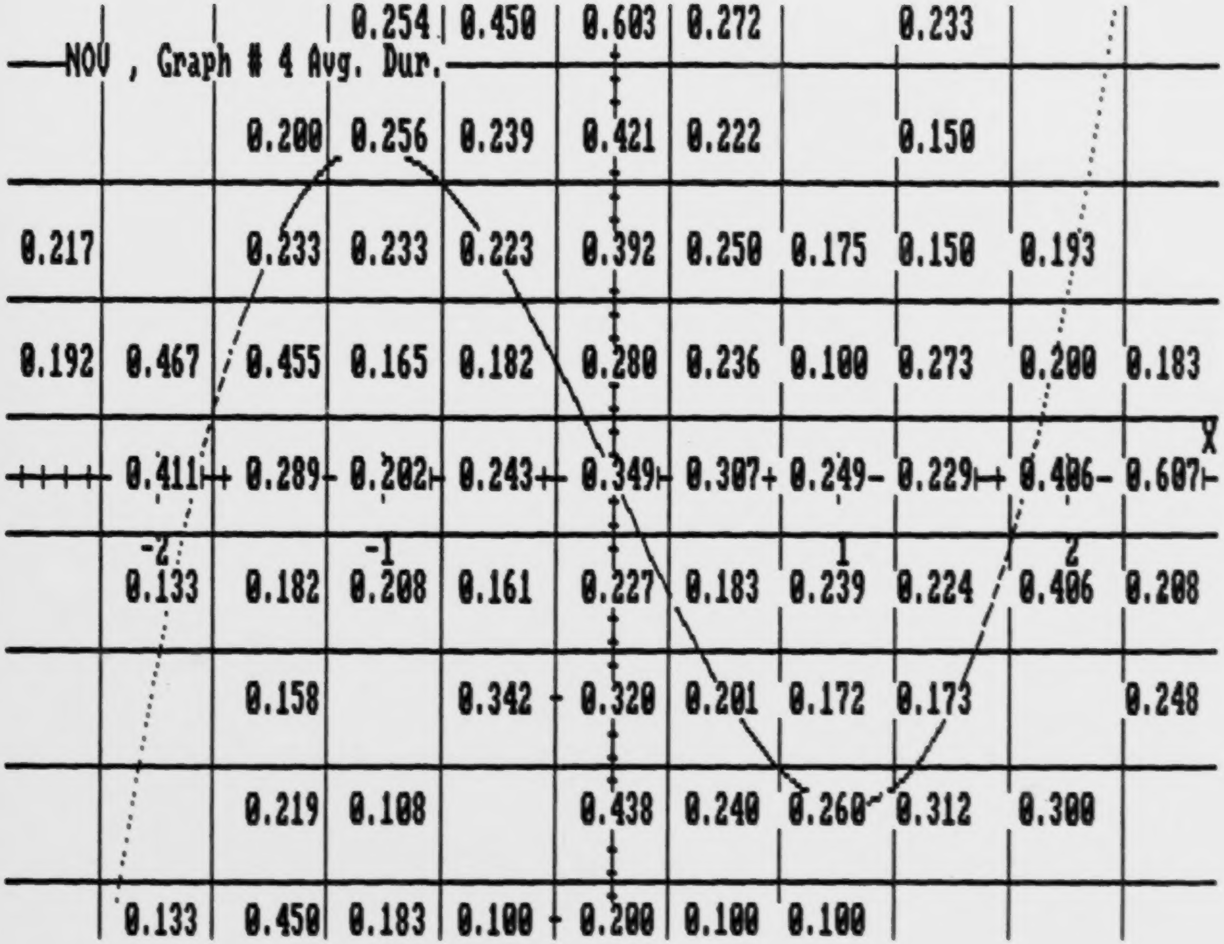


Figure 26: Scaling pattern for eye movement data with a subject's scaling data shown.



			0.477	0.106	1.416	0.192		0.055		
		0.047	1.021	1.123	3.853	0.156		0.035		
0.051		0.055	1.146	1.987	5.699	0.117	0.082	0.035	0.227	
0.180	2.852	1.815	0.657	0.814	3.086	0.332	0.070	0.321	0.141	0.086
	6.071	7.131	3.595	1.424	14.434	2.093	3.446	3.329	5.152	0.712
	0.063	0.426	0.536	0.113	1.385	0.430	2.022	1.475	5.527	0.196
		0.149		0.160	2.402	0.802	0.485	0.203		0.465
		0.618	0.051		2.981	0.900	1.705	0.512	0.070	
	0.031	0.422	0.086	0.047	0.047	0.047	0.023			

Figure 27: Map showing percent of total time for graph #4.



			0.254	0.450	0.603	0.272		0.233		
		0.200	0.256	0.239	0.421	0.222		0.150		
0.217		0.233	0.233	0.223	0.392	0.250	0.175	0.150	0.193	
0.192	0.467	0.455	0.165	0.182	0.280	0.236	0.100	0.273	0.200	0.183
	0.411	0.289	0.202	0.243	0.349	0.307	0.249	0.229	0.406	0.607
	0.133	0.182	0.208	0.161	0.227	0.183	0.239	0.224	0.406	0.208
		0.158		0.342	0.320	0.201	0.172	0.173		0.248
		0.219	0.108		0.438	0.240	0.260	0.312	0.300	
	0.133	0.450	0.183	0.100	0.200	0.100	0.100			

Figure 28: Map showing average fixation duration for graph #4.

APPENDIX B

FIGURES 29 THROUGH 48 REFERRED TO IN CHAPTER IV

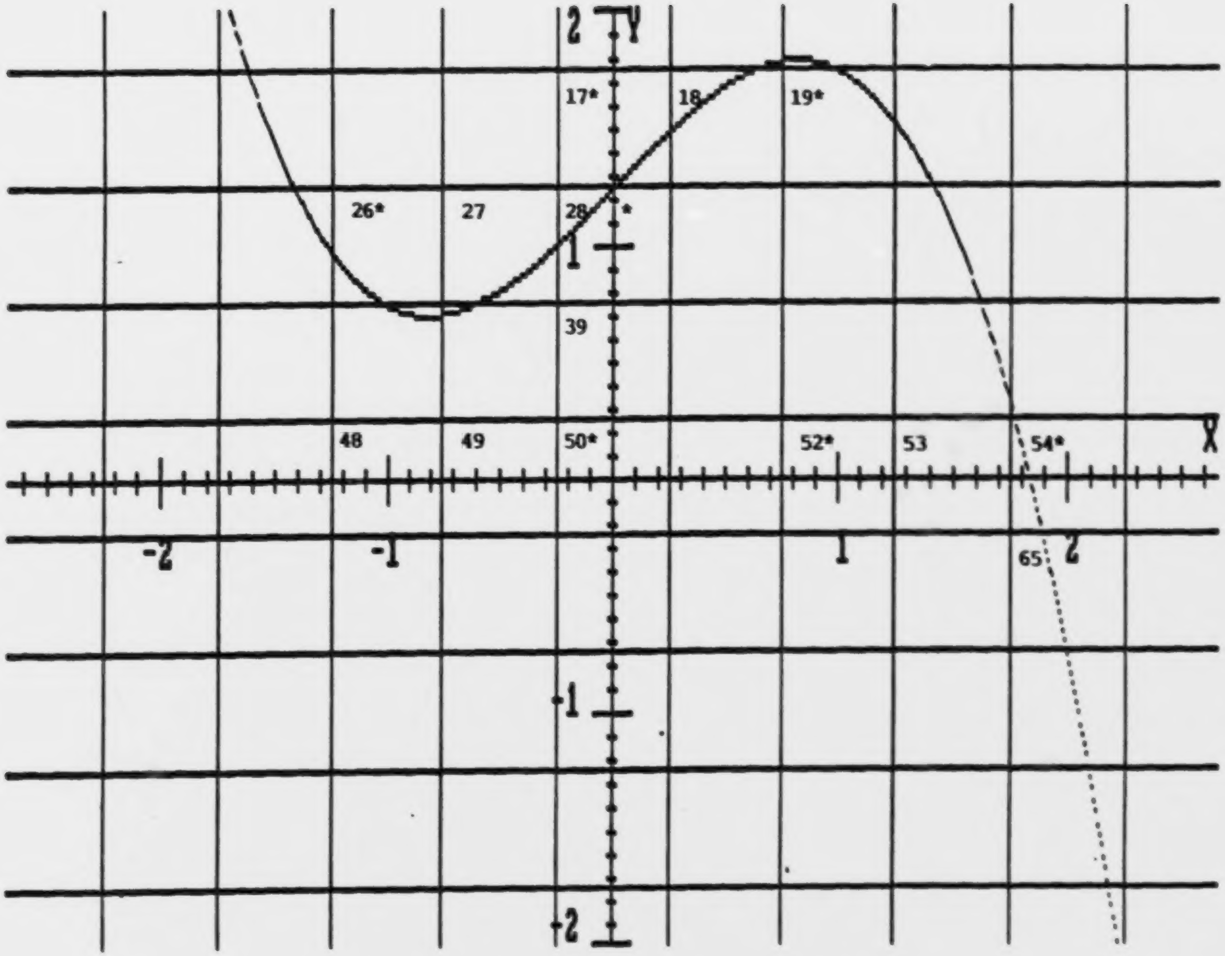


Figure 29: Graph #2 showing important and less important blocks.

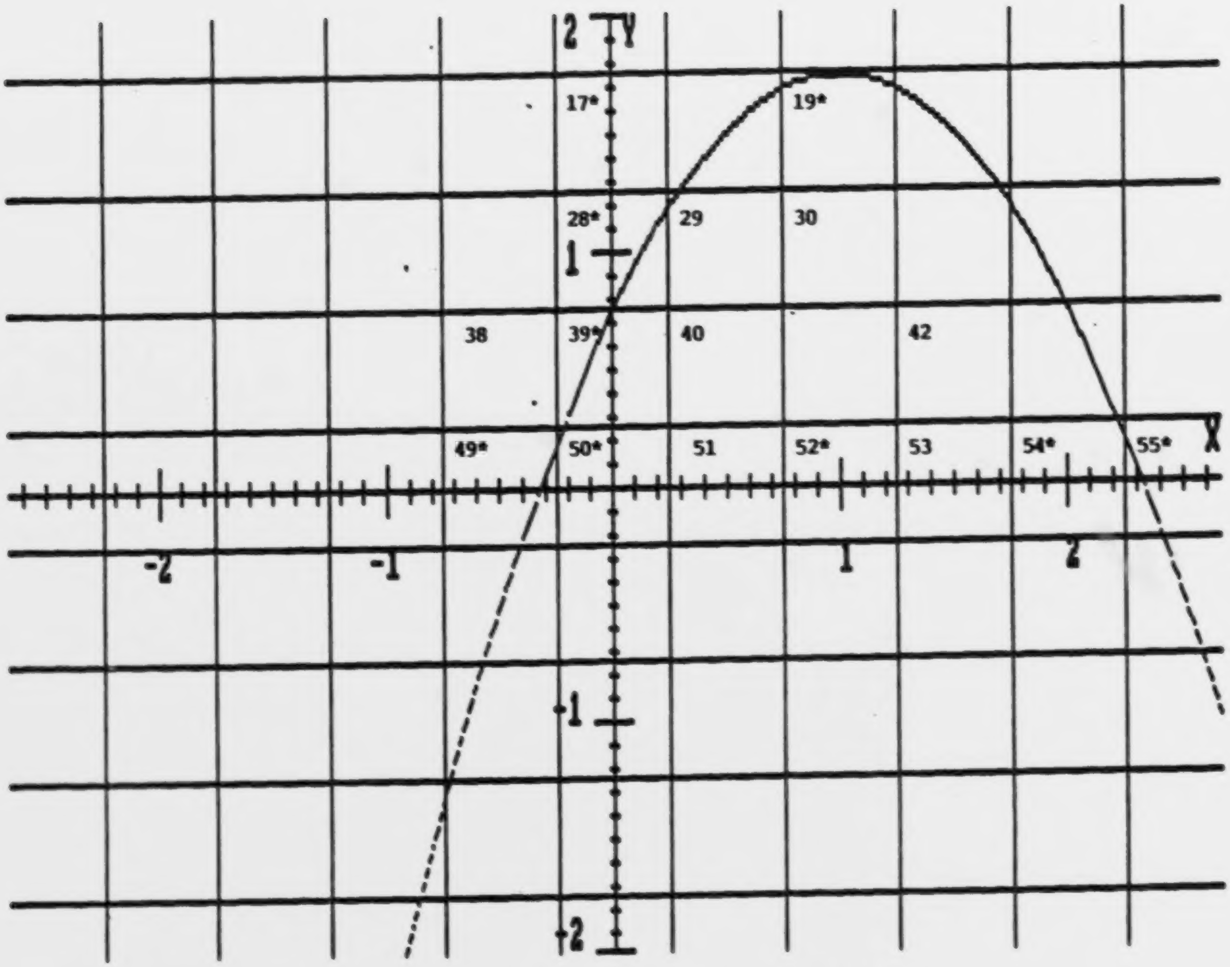


Figure 30: Graph #3 showing important and less important blocks.

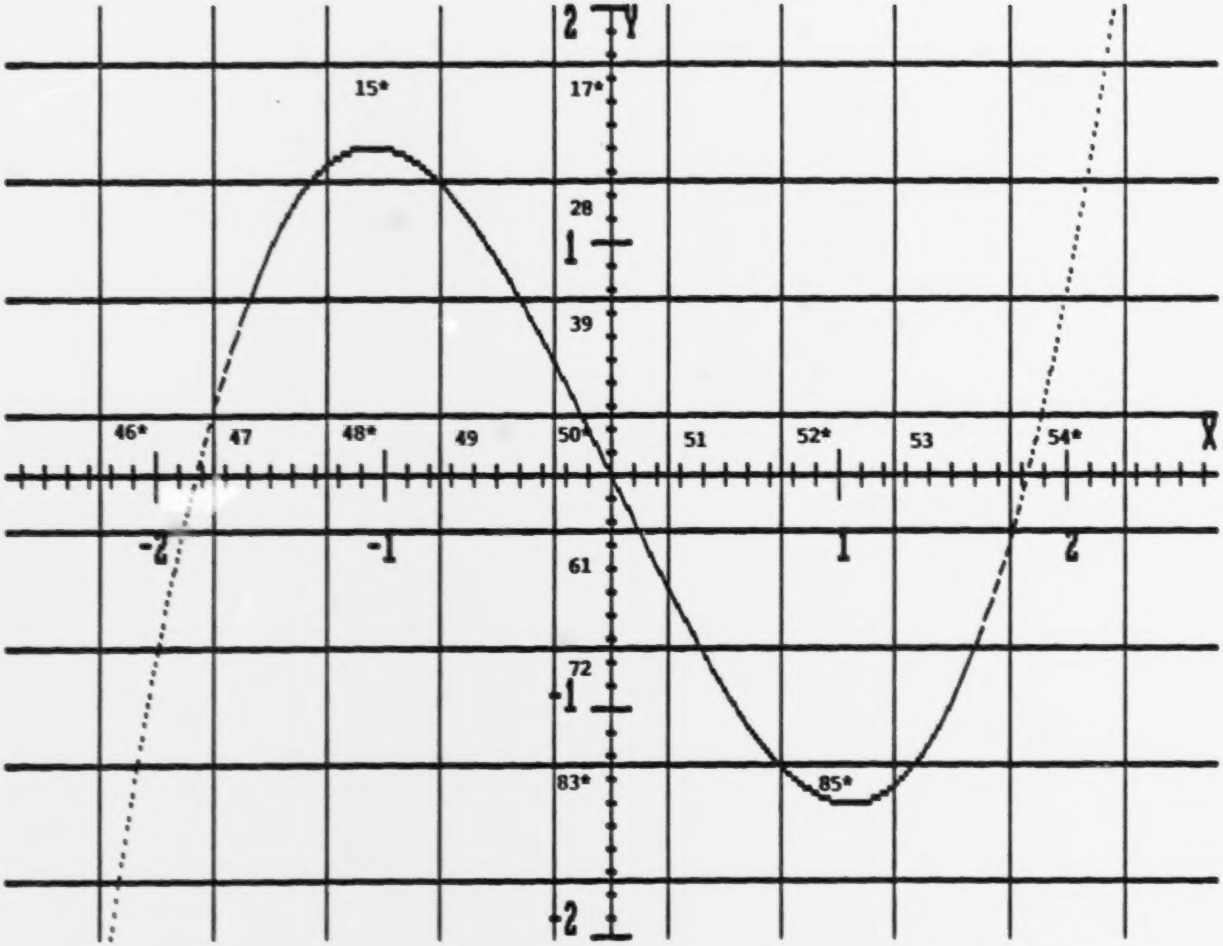


Figure 31: Graph #4 showing important and less important blocks.

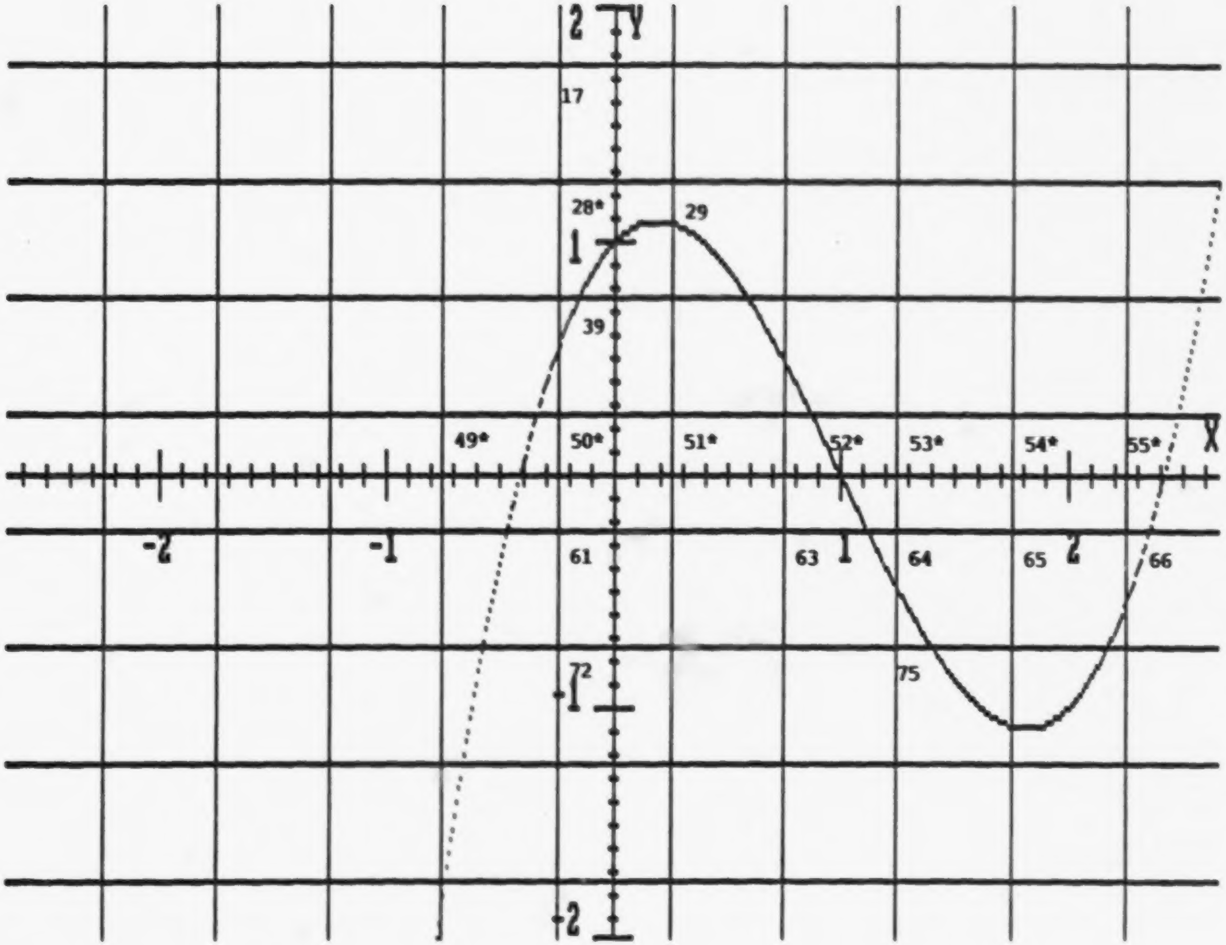


Figure 32: Graph #6 showing important and less important blocks.

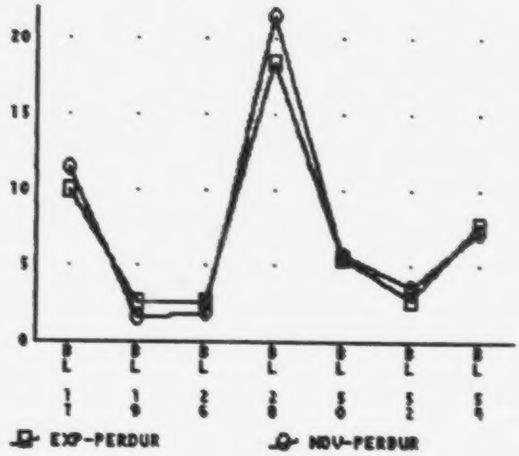


Figure 33: Plot of cell means for percent of total time of the important blocks of graph #2.

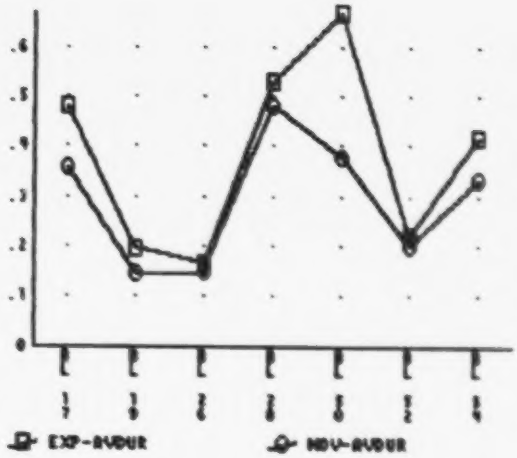


Figure 34: Plot of cell means for average fixation duration of the important blocks of graph #2.

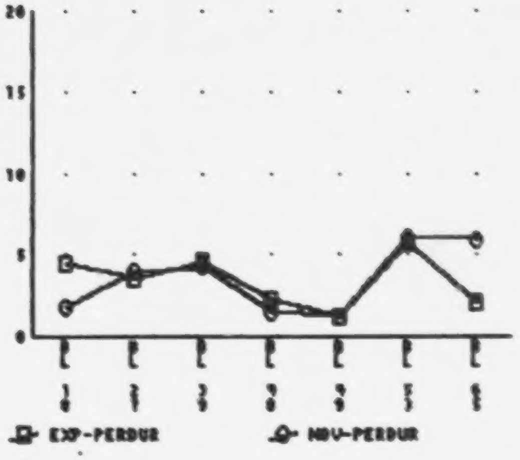


Figure 35: Plot of cell means for percent of total time for the less important blocks of graph #2.

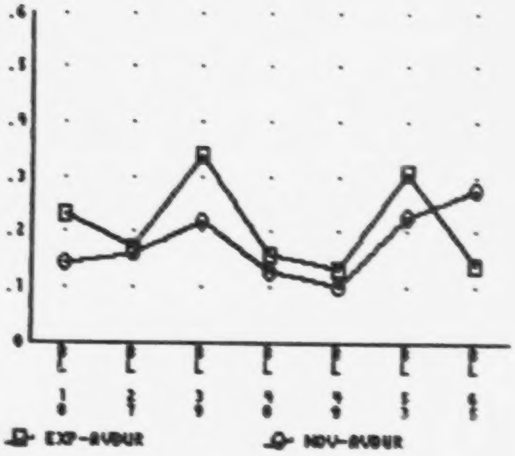


Figure 36: Plot of cell means for average fixation duration of the less important blocks of Graph #2.

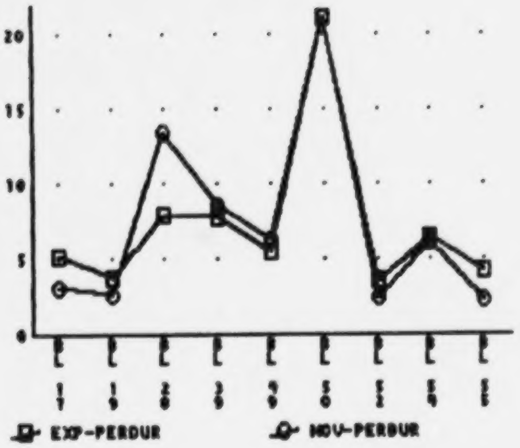


Figure 37: Plot of cell means for percent of total time of the important blocks of graph #3.

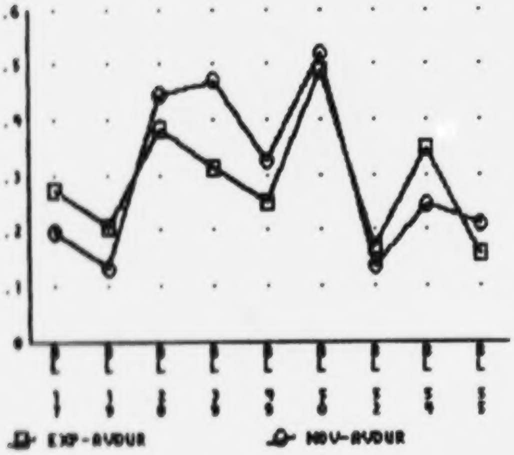


Figure 38: Plot of cell means for average fixation duration of the important blocks of graph #3.

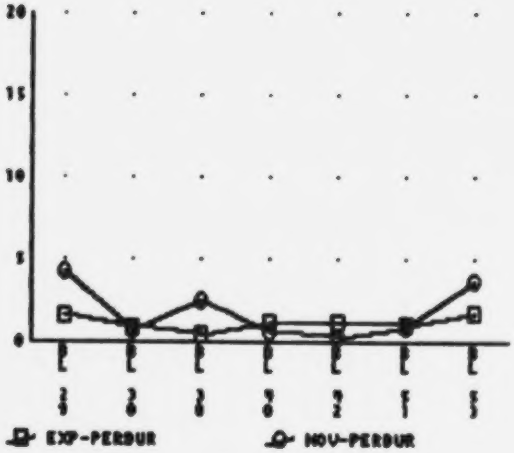


Figure 39: Plot of cell means for percent of total time for the less important blocks of graph #3.

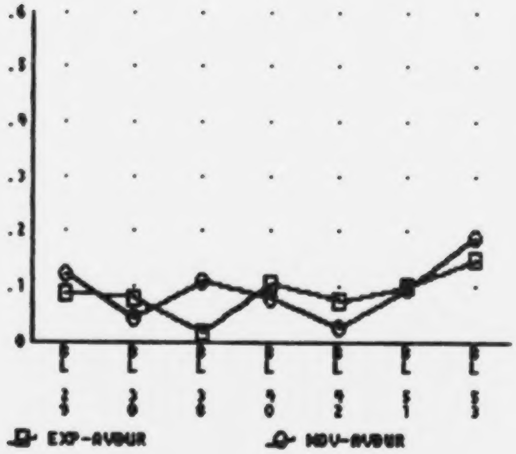


Figure 40: Plot of cell means for average fixation duration of the less important blocks of Graph #3.

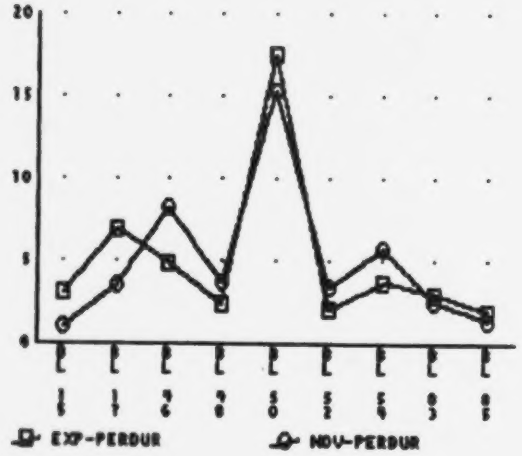


Figure 41: Plot of cell means for percent of total time of the important blocks of graph #4.

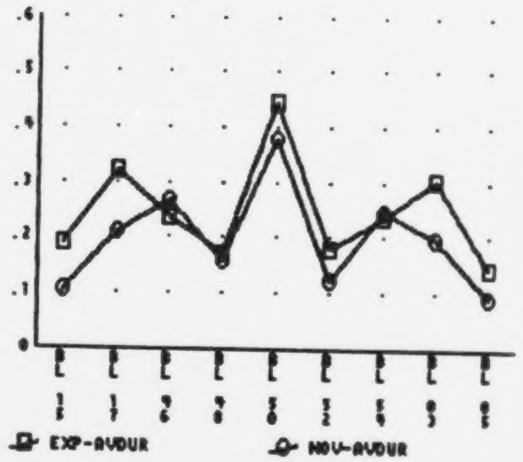


Figure 42: Plot of cell means for average fixation duration of the important blocks of graph #4.

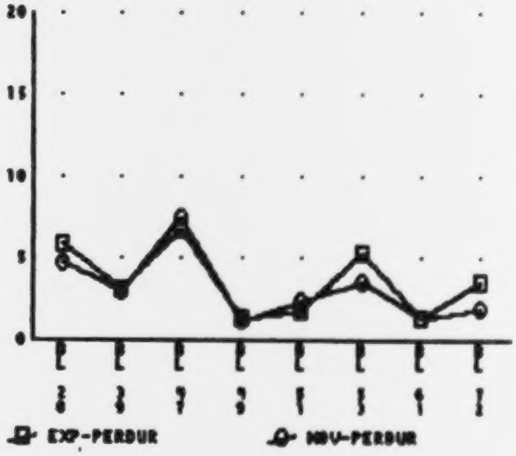


Figure 43: Plot of cell means for percent of total time for the less important blocks of graph #4.

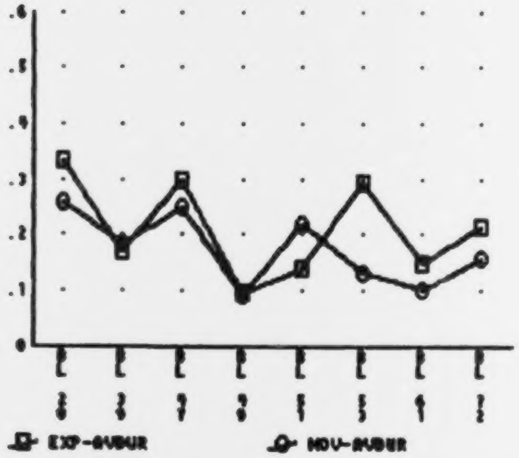


Figure 44: Plot of cell means for average fixation duration of the less important blocks of Graph #4.

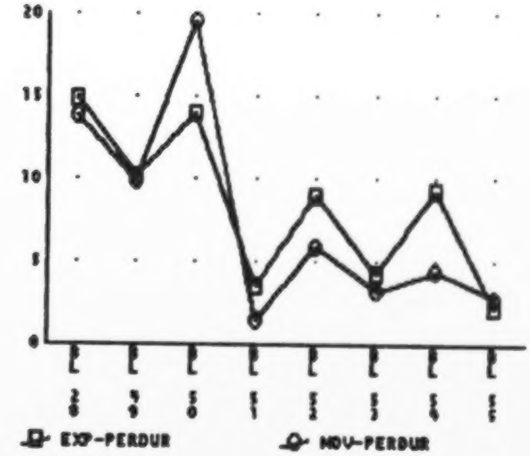


Figure 45: Plot of cell means for percent of total time of the important blocks of graph #6.

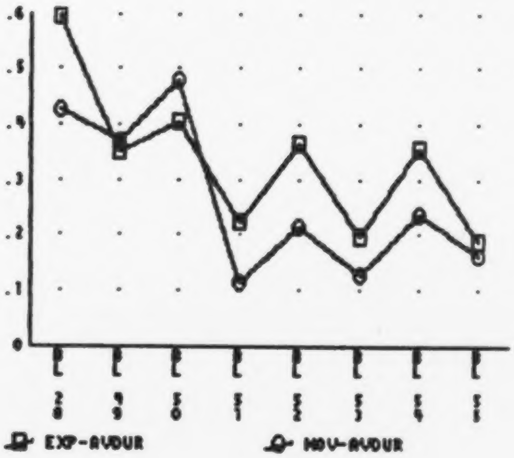


Figure 46: Plot of cell means for average fixation duration of the important blocks of graph #6.

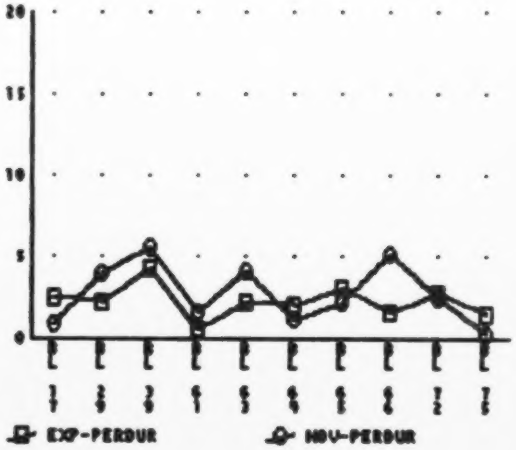


Figure 47: Plot of cell means for percent of total time for the less important blocks of graph #6.

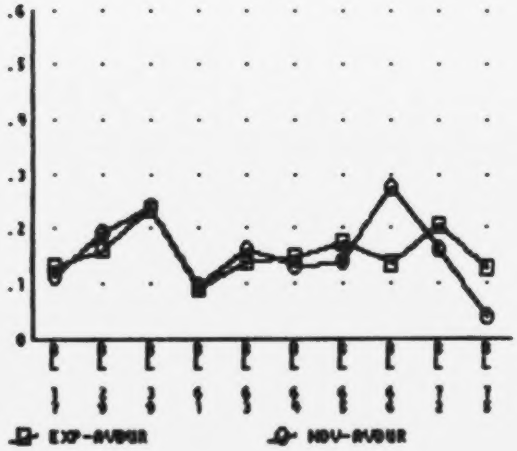


Figure 48: Plot of cell means for average fixation duration of the less important blocks of Graph #6.

APPENDIX C

TABLES 11 THROUGH 18 REFERRED TO IN CHAPTER IV

Table 11

Cell Means and Differences for Average Fixation Duration and Percent of Total Time for *Important* Blocks of Graph #2

	Average Fixation Duration			Percent Total Time		
	Exp.	Nov.	Diff.	Exp.	Nov.	Diff.
Block 17	.478	.358	+.120	9.975	11.468	-1.493
Block 19	.194	.145	+.049	2.430	1.481	+0.949
Block 26	.163	.149	+.014	2.563	1.819	+0.744
Block 28	.528	.478	+.050	18.428	21.444	-3.016
Block 50	.666	.379	+.287	5.381	5.519	-0.201
Block 52	.218	.198	+.020	2.746	3.474	-0.728
Block 54	.414	.331	+.083	7.632	7.271	+0.361

Table 12

Cell Means and Differences for Average Fixation Duration and Percent of Total Time for *Less Important* Blocks of Graph #2

	Average Fixation Duration			Percent Total Time		
	Exp.	Nov.	Diff.	Exp.	Nov.	Diff.
Block 18	.233	.142	+.091	4.362	1.746	+2.616
Block 27	.168	.162	+.006	3.446	3.905	-0.459
Block 39	.337	.219	+.118	4.491	4.154	+0.337
Block 48	.158	.126	+.032	2.128	1.425	+0.703
Block 49	.131	.099	+.032	1.133	1.071	+0.062
Block 53	.306	.227	+.079	5.695	5.938	-0.243
Block 65	.138	.274	-.136	2.042	5.838	-3.796

Table 13

Cell Means and Differences
for Average Fixation Duration and Percent of Total Time
for *Important* Blocks of Graph #3

	Average Fixation Duration			Percent Total Time		
	Exp.	Nov.	Diff.	Exp.	Nov.	Diff.
Block 17	.271	.197	+.074	5.145	2.949	+2.196
Block 19	.206	.129	+.077	3.663	2.494	+1.169
Block 28	.381	.442	-.061	7.876	13.310	-5.434
Block 39	.310	.468	-.158	7.690	8.498	-0.808
Block 49	.248	.323	-.075	5.471	6.190	-0.719
Block 50	.487	.519	-.032	21.143	21.170	-0.027
Block 52	.170	.135	+.035	3.552	2.271	+1.281
Block 54	.346	.246	+.100	6.426	6.082	+0.344
Block 55	.156	.210	-.054	4.182	2.135	+2.047

Table 14

Cell Means and Differences
for Average Fixation Duration and Percent of Total Time
for *Less Important* Blocks of Graph #3

	Average Fixation Duration			Percent Total Time		
	Exp.	Nov.	Diff.	Exp.	Nov.	Diff.
Block 29	.088	.121	-.033	1.589	4.157	-2.568
Block 30	.076	.044	+.032	0.766	0.691	+0.075
Block 38	.015	.108	-.093	0.439	2.458	-2.019
Block 40	.104	.078	+.026	1.058	0.528	+0.530
Block 42	.075	.026	+.049	1.167	0.198	+0.969
Block 51	.098	.095	+.003	0.914	0.764	+0.150
Block 53	.149	.186	-.037	1.667	3.594	-1.927

Table 15

Cell Means and Differences for Average Fixation Duration and Percent of Total Time for Important Blocks of Graph #4

	Average Fixation Duration			Percent Total Time		
	Exp.	Nov.	Diff.	Exp.	Nov.	Diff.
Block 15	.189	.102	+.087	3.021	1.023	+1.998
Block 17	.318	.211	+.107	6.788	3.525	+3.263
Block 46	.237	.268	-.031	4.739	8.223	-3.484
Block 48	.172	.157	+.015	2.216	3.649	-1.433
Block 50	.442	.379	+.063	17.475	15.199	+2.276
Block 52	.179	.122	+.057	2.045	3.315	-1.270
Block 54	.237	.245	-.008	3.650	5.641	-1.991
Block 83	.303	.196	+.107	2.896	2.466	+0.430
Block 85	.145	.092	+.053	1.797	1.303	+0.494

Table 16

Cell Means and Differences for Average Fixation Duration and Percent of Total Time for Less Important Blocks of Graph #4

	Average Fixation Duration			Percent Total Time		
	Exp.	Nov.	Diff.	Exp.	Nov.	Diff.
Block 28	.333	.259	+.074	5.753	4.636	+1.117
Block 39	.171	.184	-.013	3.058	2.861	+0.197
Block 47	.300	.250	+.050	6.668	7.491	-0.823
Block 49	.096	.091	+.005	1.266	1.103	+0.163
Block 51	.138	.218	-.080	1.664	2.234	-0.570
Block 53	.293	.131	+.162	5.260	3.483	+1.777
Block 61	.149	.100	+.049	1.264	1.273	-0.009
Block 72	.213	.157	+.056	3.425	1.885	+1.540

Table 17

Cell Means and Differences for Average Fixation Duration and Percent of Total Time for Important Blocks of Graph #6

	Average Fixation Duration			Percent Total Time		
	Exp.	Nov.	Diff.	Exp.	Nov.	Diff.
Block 28	.593	.427	+.166	14.819	13.792	+1.027
Block 49	.352	.369	-.017	10.036	9.741	+0.295
Block 50	.405	.479	-.074	13.924	19.515	-5.591
Block 51	.221	.113	+.108	3.512	1.325	+2.187
Block 52	.363	.216	+.147	8.964	5.846	+3.118
Block 53	.198	.127	+.071	4.186	3.199	+0.987
Block 54	.356	.234	+.122	9.138	4.277	+4.911
Block 55	.186	.162	+.024	2.158	2.732	-0.574

Table 18

Cell Means and Differences for Average Fixation Duration and Percent of Total Time for Less Important Blocks of Graph #6

	Average Fixation Duration			Percent Total Time		
	Exp.	Nov.	Diff.	Exp.	Nov.	Diff.
Block 17	.128	.113	+.015	2.443	0.882	+1.561
Block 29	.163	.191	-.028	2.189	3.950	-1.761
Block 39	.235	.240	-.005	4.233	5.456	-1.223
Block 61	.090	.095	-.005	0.574	1.523	-0.949
Block 63	.141	.159	-.018	2.072	4.092	-2.020
Block 64	.150	.130	+.020	1.942	1.169	+0.773
Block 65	.174	.137	+.037	3.000	2.202	+1.571
Block 66	.136	.274	-.138	1.601	5.016	-3.415
Block 72	.207	.159	+.048	2.705	2.434	+0.271
Block 75	.127	.037	+.090	1.412	0.169	+1.243

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